

Electronic transport in ferromagnetic conductors with inhomogeneous magnetic order parameter and domain-wall resistance

Christian Wickles* and Wolfgang Belzig†

Fachbereich Physik, Universität Konstanz, 78457 Konstanz, Germany

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We microscopically derive transport equations for the conduction electrons in ferromagnetic materials with an inhomogeneous magnetization profile. Our quantum kinetic approach includes elastic scattering and anisotropic spin-flip scattering at magnetic impurities. In the diffusive limit, we calculate the resistance through a domain wall and find that the domain-wall resistance can be positive or negative. In the limit of long domain walls we derive analytical expressions and compare them with existing works, which used less general models or different theoretical frameworks.

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I. INTRODUCTION

Conducting magnetic materials are an active research topic at present due to promising applications such as magnetic memory storage devices which make use of magnetization reversal in pillar multilayer nanostructures^{1–5} or domain-wall motion^{6–11} as proposed for the racetrack memory.¹² On one hand, domain-wall motion is realized by sending spin-polarized current through the domain wall, so that the mutual interaction of the electron spin with the ferromagnetic order parameter leads to a motion of the wall. This is due to the so-called spin torque,^{1,13–16} the transfer of spin-angular momentum. On the other hand, the electronic current flow is also affected by the presence of an inhomogeneous magnetization. Most prominently, there is a change in the resistance when the current runs through a domain wall in comparison to the resistance in the absence of the domain wall. The resistance change can have different origins that can be separated into the *extrinsic* and *intrinsic* domain-wall resistance (DWR). The former includes orbital and anisotropic magnetoresistance. The latter contains the direct influence the domain wall has on the electronic conduction channels: if the magnetization direction is not homogeneous in space, the spin majority and minority channels are no longer eigenstates, which in turn changes the conduction properties and also can have influence on the impurity scattering rates. There is also spin accumulation in the vicinity of the domain wall which leads to an additional potential drop. In any case, the *extrinsic* mechanisms have to be carefully identified in order to obtain the intrinsic domain-wall resistance from experiment. The DWR has been studied in a number of works in the past, both theoretically^{17–25} and experimentally.^{26–29} Reviews about DWR in nanowires made from ferromagnetic transition metals, experimental measurements, and details on the treatment of extrinsic magnetoresistance can be found in Refs. 30 and 31.

On the theoretical side, two major research lines to magnetotransport have been followed in the last years. One line is a numerical approach using atomistic so-called first principle calculation to treat relatively small systems. The second line of research applicable to larger systems, in which structural details due to a microscopic disorder averages out, is based on analytical kinetic equation derived from field theo-

retical methods. We now briefly summarize a few main results obtained with both approaches.

Using numerical methods based on *ab initio* approaches in combination with tight-binding transport calculations detailed predictions for the linear conductance of multilayers have been made.^{32–34} These include in particular all effects of the complex band structure and can also take into account the effect of disorder. The effects of a finite cross section were explored in Ref. 35 and an inhomogeneous magnetization has been accounted for in Ref. 36. A method to overcome the linear response limit of these calculations has been recently proposed for nonmagnetic materials.³⁷ These approaches are well suitable and very successful to describe multilayers and atomic size domain walls. The main disadvantage is that the computational effort limits the system size rather drastically and typical mesoscale domain walls with dimensions of the order of 10 nm cannot be described. Furthermore, spin-flip scattering by magnetic impurities or disordered spin-orbit scattering are difficult to take into account by the microscopic tight-binding simulations. Since the latter mentioned systems will be the main focus of our work, we will therefore choose a different approach, based on analytical transport equations using nonequilibrium Green's function methods.

Using analytical methods, several limiting cases of the domain-wall resistance have been investigated using a variety of theoretical approaches. The works^{18–20} perform a diagrammatic evaluation of the Kubo-formula introducing scattering in the unperturbed Green's functions by two phenomenological parameters $\tau_{\uparrow\downarrow}$, the momentum scattering times for spin-up and spin-down channels. In this calculation, spin-flip processes are not included. As we will discuss later, this leads to a spin accumulation that does not decay even arbitrarily far from the domain wall. Hence, this neglect of spin-flip is only possible, if the distance between the domain wall and leads is much smaller than the spin-diffusion length. In a complementary approach, Levy and Zhang¹⁷ use a linearized Boltzmann equation. They do not consider changes in the electronic spectrum, i.e., they assume spin-independence of the wave vector $k_{\uparrow}=k_{\downarrow}$, restricting the validity to the regime of small exchange splitting. Their analytical calculation is done in a basis that diagonalizes the Hamiltonian, which is possible in case of a constant magnetization

gradient, known as spin spiral. Thus, they cannot take into account a finite contact geometry and finite domain-wall length, but have to consider an infinitely extended spin-spiral for which they calculate the conductivity. Spin-flip processes are absent, so, again, the above statement concerning spin accumulation applies. Furthermore, they perform a multipole expansion of the distribution function but only include terms up to the p -wave component. However, as we will see during our calculation, this is not sufficient in general. Lastly, we believe the Boltzmann equation, they use, lacks terms that should appear as a result of the gauge transformation. Bergeret *et al.* use the Keldysh technique to derive a quasiclassical equation valid in the diffusive limit.²³ However, they consider a different regime of validity, in which the scattering mean-free path l_s is the smallest length scale in the system (besides the Fermi length), and not the precession length as will be the case in our treatment. Likewise, they do not consider spin-flip processes, even though during their calculation, they perform steps which implicitly require longitudinal spin excitations to relax. Finally, Simanek *et al.*^{24,25} used equations of motion for the quantum distribution function in Wigner space, which however contain a term that we cannot reproduce. Before, Bergeret *et al.* noted that this term violates particle conservation.²³ Nevertheless, this term does not affect the statement of Simanek *et al.* that there is quenching of the spin-accumulation due to rapid transverse precession. This also emerges from our theory and we will make use of it later [see the discussion around Eq. (61)].

In this paper, we pursue a fully microscopical theoretical approach to the DWR in the limit of wide domain walls, so that quantum mechanical electron reflection at the domain wall can be neglected. This allows us to use a standard quasiclassical approximation and neglect spin-dependent scattering due to abrupt potential changes.^{38–41} We use the model with quasifree electrons instead of a realistic band structure, since we are mainly interested in linear response properties, which limit the relevant energy scales to small band around the Fermi level. We assume a width of the system larger than the mean-free path, so that quantization effects due to a finite cross section do not interfere with the intrinsic DWR. Furthermore, we assume that the precession length should be much smaller than other typical lengths of the system. As opposed to other works, our approach includes spin-flip scattering, which is needed for a decay of the spin-accumulation away from the domain wall. Scattering is included on microscopic level and we include gradient corrections to the collision integral.

In Sec. II, we begin by introducing our model and deriving a quantum transport equation using the Keldysh kinetic equation approach. These provide a rather general framework to investigate a large variety of static transport problems. In Sec. III we solve these resulting equations analytically in certain limiting cases for model domain walls and discuss our results and relate them to various existing theoretical works dealing with the issue of DWR.^{17–25} Finally we conclude with an outlook on open problems.

II. QUANTUM TRANSPORT EQUATION FOR FERROMAGNETIC CONDUCTORS

In this section we derive a quantum transport equation from a model Hamiltonian that describes the kinetics of con-

duction electrons in materials with inhomogeneous magnetization profile.

A. Model and Hamiltonian

We consider a system of effectively noninteracting electrons whose spin degrees of freedom are coupled to the ferromagnetic order parameter in the mean-field approximation via the spatially dependent exchange field. The single particle Hamiltonian has three contributions,

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_S + \hat{\mathcal{V}} = \left(\frac{\hbar^2 \mathbf{k}^2}{2M} - \delta\mu(\mathbf{r}) - e\varphi(\mathbf{r}) \right) \hat{1} - \frac{\Delta}{2} \mathbf{m}(\mathbf{r}) \hat{\boldsymbol{\sigma}} + \hat{\mathcal{V}}(\mathbf{r}). \quad (1)$$

Here, \mathcal{H}_0 is the usual free quasiparticle energy contribution with the dispersion relation $\epsilon_k \equiv \frac{\hbar^2 k^2}{2M}$ and effective mass M and in spatial representation, $\mathbf{k} = -i\nabla_{\mathbf{r}}$. $\varphi(\mathbf{r})$ is the external electric potential felt by the quasiparticles of charge $-e$. $\delta\mu$ is a shift in the chemical potential due to the magnetization gradient $\partial_i \mathbf{m}$ which later turns out to be of order $\delta\mu = O(\partial_i \mathbf{m})^2$. \mathcal{H}_S describes the coupling of the electron spin to the exchange field with a constant magnitude Δ and the local magnetization direction denoted by the unit vector $\mathbf{m}(\mathbf{r})$. The 2×2 matrix spin structure is denoted by $\hat{\boldsymbol{\sigma}} = (\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3)$ is the vector of Pauli matrices, such that the electron spin operator is given by $\hat{\mathbf{S}} = \frac{\hbar}{2} \hat{\boldsymbol{\sigma}}$.

In accordance with the mean-field approach, the length scale of the spatial variations is much smaller than the relevant atomic scales. More specifically, this condition reads

$$1/l_m \equiv |\partial_i \mathbf{m}| \ll p_F/\hbar. \quad (2)$$

The exchange field is created by electrons that align their spin preferably in the same direction due to the (here ferromagnetic) exchange interaction. In conducting ferromagnets, the electrons contributing to the local magnetization can either be localized and, thus, do not participate in transport (d -electron character) or be delocalized and, hence, are subject to electronic transport phenomena (dominant s -electron character). These extreme cases constitute two distinct models with the major difference being the way in which the self-consistency condition for the exchange field is employed. These are known as s - d model and *itinerant* Stoner model, the latter one describing a system in which transport and magnetism arise in fact both from the same delocalized electrons. However, real physical systems are usually between these two cases. Below, we will restrict ourselves to the s - d model in which the magnetization profile remains static even if the conduction electrons are in a nonequilibrium configuration. Note that fluctuations of the order parameter are neglected.

We disregard the influence of the effective magnetic exchange field on the electronic orbits, which represents the Lorentz force and leads to the orbital magnetoresistance (OMR). Theoretically, as well as experimentally, the OMR and other extrinsic contributions such as the anisotropic magnetoresistance (AMR), which stems from spin-orbit coupling and leads to a resistance depending on the angle be-

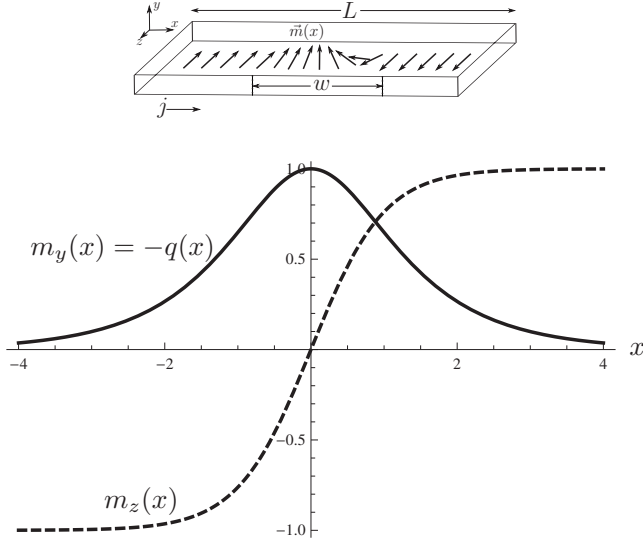


FIG. 1. Contact of length L with a Bloch domain wall of characteristic size w situated in between. The domain wall is described by the magnetization gradient $q(x) = \partial\theta(x)$. The lower plot shows $m_y(x) = -q(x)$ and $m_z(x)$ according to Eq. (38) with unity wall width $w=1$.

tween magnetization and current directions, can be separated from the true DWR and therefore are not included in this paper. For Bloch walls and the CPW geometry (current perpendicular to wall), which we employ in our model later on (see Fig. 1), the AMR plays no role, since the magnetization direction is always perpendicular to the current direction. For other walls such as Néel walls, the spin direction within the wall attains a component parallel to the current and the AMR has to be carefully distinguished from the DWR. Generally, to leading order, the AMR contribution can simply be added to the DWR. AMR is an effect of zeroth order in magnetization gradients, but since we are primarily interested in the small conductivity change imposed by the presence of a domain wall to lowest order in the domain-wall gradient, we can disregard this additional spin-orbit term. In higher order, the effects of a magnetization gradient and the mechanism underlying the AMR interfere and a more general description is necessary. Note, that in addition the anomalous Hall effect (AHE) comes into play in higher order. To stay as simple as possible, we neglect AMR and AHE effects in the following assuming they are either negligible or can be experimentally distinguished from the DWR.

The impurity scattering potential can be divided into two contributions,

$$\hat{V}(\mathbf{r}) = V_i(\mathbf{r})\hat{1} + \hat{V}_{\text{mag}}(\mathbf{r}). \quad (3)$$

\hat{V}_i describes the scattering from randomly distributed static impurities and for pointlike scatterers has the property

$$\langle V_i(\mathbf{r})V_i(\mathbf{r}') \rangle_{\text{imp}} = \chi^{(i)}\delta(\mathbf{r} - \mathbf{r}'), \quad (4)$$

equivalent to the treatment of V_i as a delta-correlated fluctuating Gaussian field. $\chi^{(i)}$ measures the strength of the impurity scattering potential and $\langle \rangle_{\text{imp}}$ denotes the averaging over all impurity configurations.

In a similar way, scattering at impurities that have internal spin degrees of freedom makes the scattering vertex spin-dependent, such that

$$\langle \hat{V}_{\text{mag}}(\mathbf{r})\hat{X}\hat{V}_{\text{mag}}(\mathbf{r}') \rangle_{\text{imp}} = \sum_{i,j=1}^3 \chi_{ij}^{(m)} \hat{\sigma}_i \hat{X} \hat{\sigma}_j \delta(\mathbf{r} - \mathbf{r}'), \quad (5)$$

where \hat{X} is an arbitrary disorder independent spin matrix. Therefore, magnetic impurity scattering in the pointlike limit can be treated as delta-correlated fluctuating Gaussian magnetic field which couples to the electron spin by the usual Zeeman term. The size of the fluctuations can generally be spin-anisotropic which manifests itself in the tensor structure of $\chi_{ij}^{(m)}$.

B. Keldysh technique

The standard way to proceed in nonequilibrium physics is to set up the kinetic equation for the Keldysh Green's function. For the remaining part of this paper, we set $\hbar=1$.

The further treatment is done in the Wigner representation which is obtained from usual spatiotemporal representation via the transformation

$$\check{G}(\mathbf{k}, \epsilon, \mathbf{r}, t) = \int d^3z d\tau e^{-ikz+i\epsilon\tau} \times \check{G}\left(\mathbf{r} + \frac{\mathbf{z}}{2}, t + \frac{\tau}{2}; \mathbf{r} - \frac{\mathbf{z}}{2}, t - \frac{\tau}{2}\right). \quad (6)$$

The product of operators has to be carried out using the Moyal product $\otimes \equiv e^{i/2(\tilde{\partial}_r \tilde{\partial}_k - \tilde{\partial}_k \tilde{\partial}_r + \tilde{\partial}_\epsilon \tilde{\partial}_t - \tilde{\partial}_t \tilde{\partial}_\epsilon)}$ where $\tilde{\partial}$ and $\tilde{\partial}$ denote derivatives acting only to the left and right, respectively.⁴²

The Green's function \check{G} is defined as expectation value of the electron field operators, time-ordered along the Keldysh contour.⁴² The ordering along backward and forward time Keldysh contour gives rise to an additional 2×2 matrix structure (denoted by $\check{}$), which in an appropriate basis takes the convenient form⁴³

$$\check{G} = \begin{pmatrix} \hat{G}^R & \hat{G}^< \\ 0 & \hat{G}^A \end{pmatrix}. \quad (7)$$

\hat{G}^R and \hat{G}^A are the retarded and advanced Green's functions, well known from equilibrium theory and carry information about the spectrum of the system. In particular, one obtains the spectral density simply from $\hat{A} = i(\hat{G}^R - \hat{G}^A)$.

The lesser component $\hat{G}^<(\mathbf{k}, \epsilon, \mathbf{r}, t)$ describes the occupation of states of the many particle system and is given in terms of electron field operators by

$$\hat{G}_{\alpha,\beta}^<(\mathbf{r}, \mathbf{r}', t, t') = i \langle \psi_\beta^\dagger(\mathbf{r}', t') \psi_\alpha(\mathbf{r}, t) \rangle, \quad (8)$$

where the grand canonical average is taken and the indices α, β are one of \uparrow, \downarrow . In equilibrium, it takes the form $\hat{G}^<(\mathbf{k}, \epsilon, \mathbf{r}) = i\hat{A}(\mathbf{k}, \epsilon, \mathbf{r})f_D(\epsilon)$ with the Fermi-Dirac distribution function $f_D(\epsilon) = 1/(e^{\beta(\epsilon - E_F)} + 1)$, where $\beta = 1/k_B T$ denotes the inverse temperature.

From the lesser Green's function $\hat{G}^<$ we can easily obtain various physical quantities of interest such as the quasiparticle spin-charge density

$$\hat{N}(\mathbf{r}, t) = -e \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi i} \hat{G}^<(\mathbf{k}, \epsilon, \mathbf{r}, t), \quad (9)$$

and spin-charge current density

$$\hat{J}(\mathbf{r}, t) = -e \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi i} \mathbf{v}_k \hat{G}^<(\mathbf{k}, \epsilon, \mathbf{r}, t), \quad (10)$$

where the quasiparticle velocity is $\mathbf{v}_k = \partial_{\mathbf{k}} \epsilon_k = \mathbf{k}/m$. The spin-charge density \hat{N} is a 2×2 matrix in spin space and its trace yields the charge density, $n_c = \text{Tr} \hat{N}$ while the spin density is given by $s = -\frac{\hbar}{2e} \text{Tr}(\hat{\sigma} \hat{N})$. In a similar manner, we obtain the charge current $j_c = \text{Tr} \hat{J}$ and the spin- k current $j_k = -\frac{\hbar}{2e} \text{Tr}(\hat{\sigma}_k \hat{J})$.

In order to find \check{G} for a specific physical system, we need an equation of motion, called the Dyson equation,

$$[(\epsilon \hat{1} - \hat{\mathcal{H}}_0 - \hat{\mathcal{H}}_S) \check{1} - \check{\Sigma}] \otimes \check{G} = \check{1}. \quad (11)$$

The self-energy $\check{\Sigma}$ incorporates scattering by magnetic and nonmagnetic impurities. The leading contribution is calculated in the self-consistent Born approximation, which truncates the series of irreducible diagrams due to multiple impurity scattering after the first one:

$$\check{\Sigma} = \text{---} \overset{\text{---}}{\text{---}} \text{---} = \langle \hat{\mathcal{V}} \check{G} \hat{\mathcal{V}} \rangle_{\text{imp}} \quad (12)$$

In physical terms, this means that all kinds of interference effects such as weak localization are dropped from the theory. In this approximation, the self-energy for spin-independent impurity scattering takes the form

$$\check{\Sigma}_i = \chi^{(i)} \int \frac{d^3k}{(2\pi)^3} \check{G}. \quad (13)$$

Similarly, for magnetic impurity scattering we obtain

$$\check{\Sigma}_{\text{mag}} = \sum_{i,j=1}^3 \chi_{ij}^{(m)} \int \frac{d^3k}{(2\pi)^3} \hat{\sigma}_i \check{G} \hat{\sigma}_j. \quad (14)$$

The tensorial structure of $\chi^{(m)}$ accounts for situations in which scattering is anisotropic in spin space. For example, if an exchange coupling between the internal impurity spin and the ferromagnetic order parameter exists, the spin is preferably aligned along this direction. Consequently, the impurity will scatter the electron with a magnitude depending on its spin. In the case of uniaxial symmetry (the symmetry axis is denoted by \mathbf{n}),

$$\chi_{ij}^{(m)} = \chi_{\perp}^{(m)} (\delta_{ij} - n_i n_j) + \chi_{\parallel}^{(m)} n_i n_j. \quad (15)$$

For the above example, the unit vector \mathbf{n} actually corresponds to the local magnetization direction \mathbf{m} . We assume throughout the rest of this paper that scattering is weak such that

$$\Sigma \ll E_F, \quad (16)$$

where E_F is the Fermi energy of the conduction electrons. Put in other words, this means that transport quantities such

as density of states do vary very slowly on the scale of the relaxation rates given by Σ . This assumption will allow us later to make use of the quasiparticle approximation.

C. Spectral density

The first step in solving the Dyson Eq. (11) is to find a solution for the spectral function. To do so, we write an equation for the retarded/advanced components of \check{G} ,

$$\left[(\epsilon - \epsilon_k + \delta\mu(\mathbf{r})) \hat{1} + \frac{\Delta}{2} \mathbf{m}(\mathbf{r}) \hat{\sigma} \right] \otimes \hat{G}^{\text{R/A}} = \hat{1}. \quad (17)$$

$\hat{G}^{\text{R/A}}$ only differs by the boundary condition which can be simply incorporated by substituting $\epsilon \rightarrow \epsilon \pm i\eta$ in Eq. (17). Furthermore, since $\Sigma \ll E_F$, we will not include impurity self energy corrections to the spectrum, i.e., we neglect the line broadening and assume a delta-peaked spectrum, commonly referred to as the quasiparticle approximation. Also, the external electric field affects the spectrum only in second order of the field.⁴³

To proceed, we perform a gradient expansion $\otimes \approx 1 + \frac{i}{2}(\tilde{\partial}_r \tilde{\partial}_k - \tilde{\partial}_i \tilde{\partial}_\epsilon - \tilde{\partial}_k \tilde{\partial}_r + \tilde{\partial}_\epsilon \tilde{\partial}_i) + \dots$, and determine $\hat{G}^{\text{R/A}}$ iteratively by the order of the gradient. It turns out, that we need only up to order ∂_r^2 . To zeroth order, one simply finds for the spectral density $\hat{A}_0 = 2\pi[\hat{P}_{\uparrow} \delta(\epsilon - \epsilon_k^{\uparrow}) + \hat{P}_{\downarrow} \delta(\epsilon - \epsilon_k^{\downarrow})]$. Here, we defined the projectors in spin-space $\hat{P}_{\uparrow, \downarrow} = \frac{1}{2}(\hat{1} \pm \mathbf{m} \hat{\sigma})$ that project on the spin-up/spin-down direction and $\epsilon_k^{\uparrow, \downarrow} \equiv \epsilon_k \mp \frac{\Delta}{2}$ is the dispersion relation for majority and minority spin bands which are exchange split due to the s - d coupling. The \mathbf{k} -integration of the spectral function yields the density of states, here defined as number of states per unit energy and volume,

$$\int \frac{d^3k}{(2\pi)^3} \hat{A}(\mathbf{k}, \epsilon, \mathbf{r}) = 2\pi \hat{\nu}(\epsilon, \mathbf{r}). \quad (18)$$

The matrix density of states without magnetization gradient present is thus

$$\hat{\nu} = \hat{P}_{\uparrow} \nu_{\uparrow} + \hat{P}_{\downarrow} \nu_{\downarrow} = \nu_0 (\hat{1} + P \mathbf{m} \hat{\sigma}), \quad (19)$$

where we have introduced the polarization of the Fermi surface

$$P = \frac{\nu_{\uparrow} - \nu_{\downarrow}}{\nu_{\uparrow} + \nu_{\downarrow}} \quad (20)$$

and $\nu_0 = \frac{1}{2}(\nu_{\uparrow} + \nu_{\downarrow})$ denotes the average density of states. The density of states for majority and minority spin bands are

$$\nu_{\uparrow, \downarrow}(\epsilon) = \int \frac{d^3k}{(2\pi)^3} \delta(\epsilon - \epsilon_k^{\uparrow, \downarrow}). \quad (21)$$

$\nu_{\uparrow, \downarrow}$ without energy argument denotes the density of states at the Fermi level.

The term $\delta\mu$ in Eq. (17) constitutes an additional chemical potential shift due to the magnetization gradient and is determined by enforcing local charge neutrality. The total quasiparticle density (9) is constant throughout the ferromagnet, i.e., $\text{Tr} \hat{N}(\mathbf{r}) = -eN_{\text{ion}}$, which leads to [see also Eq. (A7)],

$$\delta\mu(\mathbf{r}) = \frac{1}{12} \left(1 - 2P \frac{2E_F}{\Delta} \right) \frac{1}{2M} (\partial_r \mathbf{m})^2. \quad (22)$$

D. Kinetic equation for the nonequilibrium distribution

The kinetic equation for the nonequilibrium distribution function is obtained by considering the lesser component of Eq. (11),

$$-i \left[(\epsilon - \epsilon_k + \delta\mu + e\varphi) \hat{1} + \frac{\Delta}{2} \mathbf{m} \hat{\sigma} \otimes \hat{G}^< \right] = \frac{1}{2} \{ \hat{A} \otimes \hat{\Sigma}^< \} - \frac{1}{2} \{ \hat{\Gamma} \otimes \hat{G}^< \} - i [\mathfrak{R} \hat{\Sigma} \otimes \hat{G}^<] + i [\mathfrak{R} \hat{G} \otimes \hat{\Sigma}^<], \quad (23)$$

where the (anti-)commutators are defined by $\{ \hat{A} \otimes \hat{B} \} = \hat{A} \otimes \hat{B} + \hat{B} \otimes \hat{A}$ and $[\hat{A} \otimes \hat{B}] = \hat{A} \otimes \hat{B} - \hat{B} \otimes \hat{A}$. The imaginary part of the self-energy

$$\hat{\Gamma} = -2\mathcal{I} \hat{\Sigma}^R = i(\hat{\Sigma}^R - \hat{\Sigma}^A) \quad (24)$$

describes relaxation due to impurity scattering, while the real part is responsible for changes in the energy dispersion relation. There exists a Kramers-Kronig relation between real and imaginary part,

$$\mathfrak{R} \hat{\Sigma}(\epsilon) = \frac{1}{2} (\hat{\Sigma}^R + \hat{\Sigma}^A) = \frac{1}{2\pi} \mathcal{P} \int_{-\infty}^{\infty} d\epsilon' \frac{\hat{\Gamma}(\epsilon')}{\epsilon - \epsilon'}. \quad (25)$$

However, as can be seen from the integral representation in this formula, the real part depends on the complete electronic spectrum of the system, since the scattering rate is directly related to the density of states, $\Gamma(\epsilon) \propto \nu(\epsilon)$. Considering that the dynamics accompanied by a rotation of the magnetization direction, be it in time or space, is affecting only an energy region of the order of Δ , these changes constitute only a tiny fraction of the whole energy range. Thus, corrections due to magnetization dynamics to the real part can be neglected when compared to the whole background contribution, which then is just a constant (however formally diverging due to the assumption of \mathbf{k} -independent impurity scattering) and merely renormalizes the electronic spectrum. In fact, the only reason to include impurity scattering is to add momentum and spin relaxation to the conduction electron system, as contained in the imaginary part of the self-energy, $\hat{\Gamma}$. The real part can however become significant if one goes beyond the self-consistent Born approximation. Important contributions from higher order diagrams are relevant for skew and side-jump scattering which give rise to the anomalous Hall effect.^{44,45} In our case, we assume that scattering is weak and thus, there is no necessity to go beyond the first order diagram for our study. Therefore, the two commutators involving real parts can be dropped from Eq. (23).

Generally, we can distinguish the contributions to the kinetic equations emerging from two regions in energy. The electronic states deep inside the Fermi sea are affected by an inhomogeneous exchange splitting $\Delta \mathbf{m}(\mathbf{r})$. The fraction of the Fermi sea that contributes is given by Δ/E_F and not necessarily small. However, due to the Pauli principle, these

states are fully occupied for all reasonable temperatures and the change in the spectrum does not affect the dynamics of the mobile electrons close to the Fermi surface. These are responsible for the dynamics in the quantum kinetic equation, since the electrons in an energy window given by the temperature, voltage or other low-energy scales have the freedom to move. These differences can be used to eliminate the high-energy contribution from our quantum kinetic equation. Possible high-energy contribution can be simply added later once the low-energy result has been obtained, but it turns out that there are no such contributions for our case of DWR. However, if one calculates nondiagonal resistivities for two- or three-dimensional magnetization structures, there are important high energy contributions which correspond to Berry curvature effects. Furthermore, a change in the spectrum due to magnetization structures changes the Berry curvature, which can, therefore, lead to changes to the intrinsic AHE contribution. The AHE is, however, beyond the scope of this work as discussed previously. In the general case, it is therefore important to carefully check for contributions from the Fermi sea.

We perform a series of three steps to obtain an equation describing the low-energy dynamics alone. First, we eliminate the electric potential by the substitution $\omega = \epsilon + e\varphi$, which transforms the derivatives according to

$$\partial_\epsilon \rightarrow \partial_\omega, \quad \partial_r \rightarrow \partial_r - e\mathbf{E} \partial_\omega \quad (26)$$

with the electric field $\mathbf{E} = -\partial_r \varphi$.

Second, we make the ansatz

$$\hat{G}^<(\mathbf{k}, \omega, \mathbf{r}) = i\hat{A}(\mathbf{k}, \omega, \mathbf{r}) f_D(\omega - e\varphi) + \delta\hat{G}(\mathbf{k}, \omega, \mathbf{r}). \quad (27)$$

The first term drops out from the kinetic Eq. (23), leaving an equation for $\delta\hat{G}$ alone. $\delta\hat{G}$ has two very practical properties. It is proportional to the electric field, which allows us to drop the term $e\varphi(\mathbf{r})\delta\hat{G}$, since we are interested only in linear response to an external field. The external potential is incorporated via appropriate boundary conditions that are concretized below. Furthermore, $\delta\hat{G}$ is peaked around the Fermi level, reflecting the fact that nonequilibrium processes take place only in the vicinity of E_F , provided the temperature is low enough. In fact, we use the zero-temperature approximation $\frac{\partial f_D}{\partial \epsilon} = -\delta(\epsilon - E_F)$ throughout this work.

This brings us directly to the third step which consists of integrating the whole equation over energy after setting $\omega = E_F$ in all prefactors to $\delta\hat{G}$ on the right-hand side of Eq. (23). The spectral densities in the collision integral become $\hat{A}(\mathbf{k}, E_F, \mathbf{r})$, which means we neglect the energy dependence of the scattering rates. This is perfectly compatible with the linear response regime, since a more in-depth investigation shows that corrections due to the energy dependence of \hat{A} are of quadratic order in the electric field.

For the stationary situation ($\partial_t = 0$), the resulting kinetic equation for

$$\hat{g}(\mathbf{k}, \mathbf{r}) = \int \frac{d\omega}{2\pi i} \delta\hat{G}(\mathbf{k}, \omega, \mathbf{r}) \quad (28)$$

finally reads

$$\mathbf{v}_k \partial_r \hat{g} - i \frac{\Delta}{2} [\mathbf{m} \hat{\sigma} \circ \hat{g}] + (\partial_r \delta \mu) \partial_k \hat{g} = \hat{\mathcal{I}}_i[\hat{g}] + \hat{\mathcal{I}}_m[\hat{g}]. \quad (29)$$

The collision integral takes the form

$$\hat{\mathcal{I}}_i[\hat{g}] = \chi^{(i)} \int \frac{d^3 k'}{(2\pi)^3} \left[\frac{1}{2} \{ \hat{A}(\mathbf{k}, E_F, \mathbf{r}) \circ \hat{g}(\mathbf{k}', \mathbf{r}) \} - \frac{1}{2} \{ \hat{A}(\mathbf{k}', E_F, \mathbf{r}) \circ \hat{g}(\mathbf{k}, \mathbf{r}) \} \right] \quad (30)$$

and similar for the magnetic scattering contribution $\hat{\mathcal{I}}_m[\hat{g}]$. \circ denotes the Moyal product with derivatives ∂_r and ∂_k only, and does *not* act on \mathbf{k}' . Again, we will need to include only terms up to order ∂_r^2 . Equation (29) along with Eq. (30) constitutes a linear integrodifferential equation for \hat{g} and serves as the basis for our further analytical treatment.

Provided we have found a solution for \hat{g} with appropriate boundary conditions, we can finally determine the physical observables of interest by simply substituting the ansatz (27) into Eq. (9). In this way, we obtain the quasiparticle spin-charge density,

$$\hat{N}(\mathbf{r}) = -e \int \frac{d\omega}{2\pi} \int \frac{d^3 k}{(2\pi)^3} \hat{A}(\mathbf{k}, \omega, \mathbf{r}) f_D(\omega) - e^2 \varphi(\mathbf{r}) \frac{1}{2\pi} \int \frac{d^3 k}{(2\pi)^3} \hat{A}(\mathbf{k}, E_F, \mathbf{r}) + \hat{n}(\mathbf{r}) \quad (31)$$

with the low-energy spin-charge density excitations $\hat{n} = -e \int \frac{d^3 k}{(2\pi)^3} \hat{g}$. We made use of the zero-temperature approximation and smallness of the external potential in the linear regime, viz $f_D(\omega - e\varphi) = f_D(\omega) + e\varphi \delta(\omega - E_F)$. The (spin-) current is obtained in a similar straightforward manner.

On length scales much larger than the Thomas-Fermi screening length, which is of the order of atomic distances in metallic materials, local charge neutrality is fulfilled. Since the first term on the right-hand side of Eq. (31) corresponds to the equilibrium density which exactly neutralizes the material, the trace of the last two terms has to vanish,

$$\text{Tr} \hat{n}(\mathbf{r}) = e^2 \varphi(\mathbf{r}) \frac{1}{2\pi} \int \frac{d^3 k}{(2\pi)^3} \text{Tr} \hat{A}(\mathbf{k}, E_F, \mathbf{r}). \quad (32)$$

Therefore, $\text{Tr} \hat{n}(\mathbf{r})$ is directly related to the local electric potential and thus the local electric field $E(\mathbf{r}) = -\partial_r \varphi(\mathbf{r})$ in the system.

Now we consider a region far from any inhomogeneities in the magnetization for which we want to specify an appropriate boundary condition. In the absence of a magnetization gradient and nonequilibrium spin excitations in the conduction electron system, a less strict equality between the last two terms of Eq. (31) holds,

$$\hat{n}(\mathbf{r}) = e^2 \varphi(\mathbf{r}) \frac{1}{2\pi} \int \frac{d^3 k}{(2\pi)^3} \hat{A}(\mathbf{k}, E_F, \mathbf{r}). \quad (33)$$

In a simple one-dimensional geometry and at the left and right boundaries $x_{L,R}$, the boundary condition simply becomes

$$\hat{n}(x_{L,R}) = e^2 \varphi(x_{L,R}) \nu_0 (\hat{1} + P \mathbf{m} \hat{\sigma}). \quad (34)$$

Before continuing to solve the kinetic Eq. (29) for \hat{g} , let us specify a convenient geometry for the contact.

III. DOMAIN-WALL RESISTANCE IN DIFFUSIVE WIRES WITH CURRENT PERPENDICULAR TO WALL (CPW) GEOMETRY

A. Model of the contact

We treat a ferromagnet in a quasi one-dimensional geometry such that there are only gradients of the magnetization in the x direction, while in the y and z directions the system is homogeneous. Furthermore, we assume a coplanar magnetization which allows to parameterize the magnetization direction by a single angle $\theta(x)$, defined by

$$\mathbf{m}(x) = \begin{pmatrix} 0 \\ \sin \theta(x) \\ \cos \theta(x) \end{pmatrix}. \quad (35)$$

Current flow perpendicular to the domain wall implies that the direction of \mathbf{j} is along the x axis, so that the problem becomes effectively one-dimensional. Therefore, in this case, every quantity of interest only depends on x , we can set $\partial_y = \partial_z = 0$ and abbreviate $\partial \equiv \partial_x$.

The contact of length L contains a domain wall of characteristic width w . The left- and rightmost parts have opposite magnetization directions and a so far arbitrary magnetization profile $\theta(x)$ connects the two magnetic domains. The situation is sketched in Fig. 1.

It is then convenient to perform a local SU(2) gauge transformation in order to rotate the coordinate system used to represent the spin-orientation such that its z axis is always aligned to the local magnetization direction \mathbf{m} . The unitary operator ($\hat{U}^\dagger = \hat{U}^{-1}$),

$$\hat{U} = e^{-(i/2)\theta \hat{\sigma}_x} \quad (36)$$

exactly performs this rotation in spin-space using the previously introduced angle θ , so that $\hat{U} \mathbf{m} \hat{\sigma} \hat{U}^{-1} = \hat{\sigma}_z$. As θ is spatially dependent, the derivatives obtain an additional term after the transformation,

$$\hat{U} \partial \hat{U}^{-1} = \hat{1} \partial + \frac{i}{2} q(x) \hat{\sigma}_x \equiv \hat{\partial}. \quad (37)$$

Henceforth, we have to deal with the gradient $q(x) = \partial \theta$ in our equations and which fully parameterizes the domain wall. The gauge transformation (36) rotates the orthonormal basis system $(\mathbf{v} \times \mathbf{m}, \mathbf{v}, \mathbf{m})$ such that it becomes the canonical set basis vectors, $(\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z)$. The unit vector \mathbf{v} points in the direction of change of the magnetization \mathbf{m} , i.e., $\partial \mathbf{m} = q \mathbf{v}$. Note that \mathbf{v} is perpendicular to \mathbf{m} , since \mathbf{m} is a unit vector.

An analytical form of the domain-wall profile can be derived in the context of minimizing the free energy of the ferromagnetic material, commonly a compromise between exchange and anisotropy energy. Typically, one finds

$$\cos \theta(x) = \tanh\left(\frac{x}{w}\right), \quad (38)$$

so that

$$q^2(x) = \frac{1}{w^2 \cosh^2\left(\frac{x}{w}\right)} = \frac{1}{w} \frac{\partial}{\partial x} \tanh\left(\frac{x}{w}\right), \quad (39)$$

where w constitutes a typical length scale over which the domain wall extends.

B. Hierarchy of equations

In light of the solution procedure that is to come, it is convenient to use a four-component vector representation (which is denoted by an arrow to distinguish it from the three-component real-space vectors set in boldface), in which the spin-charge density excitations and current density take the form, respectively,

$$\vec{n} = (n_+, n_-, n_\uparrow, n_\downarrow),$$

$$\vec{j} = (j_+, j_-, j_\uparrow, j_\downarrow).$$

Starting from the 2×2 -matrix representation after the gauge transformation, $\hat{n} = \frac{n_c}{2} \hat{1} + n_x \hat{\sigma}_x + n_y \hat{\sigma}_y + n_z \hat{\sigma}_z$, we define the spin-up/down densities $n_{\uparrow, \downarrow} = \frac{n_c}{2} \pm n_z$. Additionally, the transverse spin degrees of freedom are transformed according to $n_\pm = n_x \pm i n_y$, which corresponds to circularly polarized transverse spin excitations. This basis is convenient because it diagonalizes the equations of motion for the transverse dynamics in absence of magnetization gradients, see Eq. (47) and following.

The full kinetic equation for \hat{g} is still very involved, so we need to perform additional simplifying steps. Therefore, we multiply Eq. (29) with $v_x^n (n \geq 0)$ and afterward integrate over the whole k space. This involves evaluating terms of the form

$$\begin{aligned} \langle v_x^n \partial_{k_x}^m \hat{g} \rangle &= \int \frac{d^3 k}{(2\pi)^3} v_x^n \partial_{k_x}^m \hat{g} = (-1)^m \int \frac{d^3 k}{(2\pi)^3} \hat{g} \partial_{k_x}^m v_x^n \\ &= \begin{cases} \frac{(-1)^m}{M^m} \frac{n!}{(n-m)!} \hat{g}^{(n-m)} & \text{if } n \geq m \\ 0 & \text{if } n < m \end{cases} \end{aligned}$$

and defining moments of the Green's function \hat{g}

$$\hat{g}^{(n)} = -e \int \frac{d^3 k}{(2\pi)^3} v_x^n \hat{g}. \quad (40)$$

The first two momenta are $\hat{g}^{(0)} = \hat{n}$ and $\hat{g}^{(1)} = \hat{j}$, obviously. The kinetic Eq. (29) turns into an infinite hierarchy of equations relating these moments. It is nevertheless possible to find an analytical solution to these equations in form of a systematic series expansion in the function $q(x)$. As we will show below, it is then possible to truncate the hierarchy by setting $\hat{g}^{(5)} = 0$ in order to calculate the domain-wall resistance. The first two of these equations are reminiscent of spin-charge

continuity and diffusion equations and take the explicit form

$$\underline{\Gamma} \vec{n} + \underline{\partial} \vec{j} = O(q), \quad (41)$$

$$\underline{\Pi} (\vec{j} + \underline{D} \partial \vec{n}) = O(q), \quad (42)$$

where we defined the derivative $\underline{\partial}$, modified by the SU(2) gauge transformation [see Eq. (37)],

$$\underline{\partial} \equiv \underline{1} \partial + q \underline{M}_x \equiv \underline{1} \partial + \frac{i}{2} q \begin{pmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix}. \quad (43)$$

The right-hand sides of these equations include important contributions that are due to modification of various transport properties in presence of a magnetization gradient like the change in the density of states which enters the collision integrals. Thus, the right-hand side vanishes as $q \rightarrow 0$. Their explicit form along with the equations for $\hat{g}^{(2)}$, etc. are rather lengthy and not needed for our following discussion. Further details can be found in the appendix in Eqs. (A17)–(A25).

The various scattering rates associated with momentum relaxation $\underline{\Pi}$ and spin-flip/-dephasing $\underline{\Gamma}$ are most conveniently expressed, introducing the parameters γ, κ and ξ , in the following way:

$$\begin{aligned} \underline{\Pi} &= \text{diag}(\gamma_j^\perp + i\Delta, \gamma_j^\perp - i\Delta, \gamma_j^\uparrow, \gamma_j^\downarrow) \\ &= \gamma_j^\perp \text{diag}\left(1 + \frac{i}{\chi}, 1 - \frac{i}{\chi}, 1 + \gamma, 1 - \gamma\right), \\ \underline{\Gamma} &= \begin{pmatrix} \gamma_n^\perp + i\Delta & 0 & 0 & 0 \\ 0 & \gamma_n^\perp - i\Delta & 0 & 0 \\ 0 & 0 & \gamma_n^\uparrow & -\gamma_n^\downarrow \\ 0 & 0 & -\gamma_n^\uparrow & \gamma_n^\downarrow \end{pmatrix} \\ &= \gamma_n^\perp \begin{pmatrix} 1 + \frac{i}{\xi\chi} & 0 & 0 & 0 \\ 0 & 1 - \frac{i}{\xi\chi} & 0 & 0 \\ 0 & 0 & \frac{1}{2}(1-P)\kappa & -\frac{1}{2}(1+P)\kappa \\ 0 & 0 & -\frac{1}{2}(1-P)\kappa & \frac{1}{2}(1+P)\kappa \end{pmatrix}. \end{aligned}$$

Spin precession is incorporated as well and manifests itself as an imaginary part in the entries of the transverse subspace of $\underline{\Gamma}$ and $\underline{\Pi}$.

In terms of the parameters of our specific model, the various scattering parameters take the form

$$\chi = \frac{\gamma_j^\perp}{\Delta} = \frac{2\pi(\nu_\uparrow + \nu_\downarrow)}{2\Delta} (\chi^{(i)} + \chi_\parallel^{(m)} + 2\chi_\perp^{(m)}),$$

$$\xi = \frac{\gamma_n^\perp}{\gamma_j^\perp} = 2 \frac{\chi_\parallel^{(m)} + \chi_\perp^{(m)}}{\chi^{(i)} + \chi_\parallel^{(m)} + 2\chi_\perp^{(m)}},$$

$$\gamma = P \frac{\chi^{(i)} + \chi_{\parallel}^{(m)} - 2\chi_{\perp}^{(m)}}{\chi^{(i)} + \chi_{\parallel}^{(m)} + 2\chi_{\perp}^{(m)}} = P(1 - \xi\kappa),$$

$$\kappa = \frac{2\chi_{\perp}^{(m)}}{\chi_{\parallel}^{(m)} + \chi_{\perp}^{(m)}}. \quad (44)$$

χ measures the ratio of momentum relaxation rate to exchange splitting Δ , ξ denotes the transverse spin-dephasing strength and γ can be considered as scattering asymmetry parameter while κ is the strength of spin-flip scattering.

Of course, the parameter γ carries an implicit dependency on the exchange splitting Δ entering through the polarization parameter

$$P(\tilde{\Delta}) = \frac{\tilde{\Delta}}{1 + \sqrt{1 - \tilde{\Delta}^2}}, \quad (45)$$

expressed here by the dimensionless exchange splitting $\tilde{\Delta} \equiv \frac{\Delta}{2E_F}$. Within our specific model of impurity scattering, $\gamma = P(1 - \xi\kappa)$ is not independent of the other parameters. It is worth mentioning that this specific form of γ has to be employed in order to guarantee for a consistent treatment of the gradient corrections in the collision integral.

Furthermore, the matrix of diffusion constants reads

$$\underline{\underline{D}} = \text{diag} \left(\frac{\chi D_{\perp}}{\chi + i}, \frac{\chi D_{\perp}}{\chi - i}, D_{\uparrow}, D_{\downarrow} \right)$$

$$= D_{\perp} \text{diag} \left(\frac{\chi}{\chi + i}, \frac{\chi}{\chi - i}, \frac{1 + \tilde{\Delta}}{1 + \gamma}, \frac{1 - \tilde{\Delta}}{1 - \gamma} \right), \quad (46)$$

where we introduced $D_{\perp} = \frac{2E_F}{3M} \frac{1}{\gamma_{\perp}^2} = \frac{1}{3} v_F^2 \frac{1}{\gamma_{\perp}^2}$. Transverse spin excitations are also subject to precession as they diffuse which results in the complex values of the effective transverse diffusion constants.

Finally, let us stress that the reason for the inclusion of the moments up to $\hat{g}^{(4)}$ lies in the fact that in our regime of investigation the precession length $l_{\text{prec}} \equiv 2\pi \frac{v_F}{\sqrt{3}\Delta}$ is, besides the Fermi length, the smallest length scale in the system. As can be seen from the Eq. (51), l_{prec} describes the period of oscillation of transverse spin excitations. In the diffusive approximation, as for example used by Bergeret *et al.*,²³ the scattering mean-free path l_s is the smallest length scale in the system (besides the Fermi wavelength) and not l_{prec} . Hence, it is possible to truncate the hierarchy of equations already by $\hat{g}^{(3)}=0$, which results in only two equations that are just the spin-charge continuity and diffusion equation. The latter is obtained by plugging the equation for $\hat{g}^{(2)}$ into the equation for $\hat{g}^{(1)}$ (see also Appendix , for more details).

C. Solving the hierarchy of equations

We eliminate the higher order moments $\hat{g}^{(n \geq 1)}$ by iteratively substituting the equations into each other, carefully keeping terms that contribute up to order q^2 . We find a differential equation for the vector of quasiparticle excitations \vec{n} of the form

$$(\underline{\underline{\Gamma}} - \underline{\underline{D}} \partial^2) \vec{n} = \underline{\underline{W}}(q) \vec{n}, \quad (47)$$

where the differential operator $\underline{\underline{W}}(q)$ vanishes for $q \rightarrow 0$ and contains all possible terms up to order q^2 . In the homogeneous ($q=0$) and collinear case, Eq. (47) corresponds to the transport equation used by Valet and Fert.⁴⁶ Let us stress that \vec{n} represents the unscreened spin-charge density excitations, while the true, screened quantity $\vec{n}^{(s)}$ is, according to Eq. (31),

$$n_{\uparrow, \downarrow}^{(s)}(x) = n_{\uparrow, \downarrow}(x) - \nu_{\uparrow, \downarrow}(x) e^2 \varphi(x). \quad (48)$$

Here, spin excitations are not screened since our model does not include a spin-dependent interaction.

Generally, $\underline{\underline{W}}(q)$ contains terms of the form $\underline{\underline{Y}}_{ijk} \partial^i q \partial^j q \partial^k$, $\underline{\underline{Y}}_{ij} \partial^i q \partial^j$, and $\underline{\underline{Y}}_i \partial^i$, where the order of q and ∂ is crucial, since ∂ acts on everything to its right. The constant matrices $\underline{\underline{Y}}$ that depend on our set of parameters, $\tilde{\Delta}$, γ , ξ , κ can be obtained in a straightforward manner from Eqs. (A17)–(A21) by collecting all terms associated with the corresponding factor $\partial^i q \partial^j q \partial^k$. Restricted to terms that contribute to DWR, its explicit form [see Eq. (A23)] is given in the appendix.

Since we have a perturbative treatment in q in mind, we determine the Green's function of Eq. (47)

$$(\underline{\underline{\Gamma}} - \underline{\underline{D}} \partial^2) \underline{\underline{G}}(x) = \underline{\underline{1}} \delta(x). \quad (49)$$

Separated into longitudinal (l) and transverse (t) subspace, the Green's function is

$$\underline{\underline{G}}(x) = \begin{pmatrix} \underline{\underline{G}}_{\text{tt}}(x) & 0 \\ 0 & \underline{\underline{G}}_{\parallel}(x) \end{pmatrix},$$

$$\underline{\underline{G}}_{\parallel}(x) = \underline{\underline{H}} e^{-|x|/\lambda} + \underline{\underline{K}} \frac{|x|}{\lambda}, \quad (50)$$

$$\underline{\underline{G}}_{\text{tt}}(x) = \begin{pmatrix} f(x) & 0 \\ 0 & f^*(x) \end{pmatrix},$$

$$f(x) = \frac{\chi + i}{\chi D_{\perp}} \frac{i}{2k} e^{ik|x|},$$

$$k^2 = \left(\frac{2\pi}{l_{\text{prec}}} \right)^2 (1 - i\chi)(1 - i\chi\xi). \quad (51)$$

Matrices in the 2×2 subspaces are denoted by a single underbar as compared to the double underbar which indicates a 4×4 matrix. The index l and t refers to longitudinal and transverse components, respectively, so that for example

$$\underline{\underline{W}} = \begin{pmatrix} \underline{\underline{W}}_{\text{tt}} & \underline{\underline{W}}_{\text{tl}} \\ \underline{\underline{W}}_{\text{lt}} & \underline{\underline{W}}_{\parallel} \end{pmatrix}.$$

The longitudinal component $\underline{\underline{G}}_{\parallel}$ consists of two contributions. The first term of $\underline{\underline{G}}_{\parallel}$ describes spatial damping of spin-up/down nonequilibrium excitations which manifests itself in the characteristic exponential decay on the spin-diffusion length,

$$\frac{1}{\lambda^2} \equiv \frac{\gamma_n^\uparrow}{D_\uparrow} + \frac{\gamma_n^\downarrow}{D_\downarrow}. \quad (52)$$

The second term of $\underline{G}_{\parallel}$ describes the linear behavior of the chemical potential in a homogeneous system in the absence of a magnetization gradient. The two tensors

$$\underline{H} = \frac{1}{2} \frac{\lambda^3}{(D_\uparrow D_\downarrow)^2} \begin{pmatrix} D_\downarrow^2 \gamma_n^\uparrow & -D_\downarrow D_\uparrow \gamma_n^\uparrow \\ -D_\downarrow D_\uparrow \gamma_n^\uparrow & D_\uparrow^2 \gamma_n^\uparrow \end{pmatrix}, \quad (53)$$

$$\underline{K} = -\frac{1}{2} \frac{\lambda^3}{D_\uparrow D_\downarrow} \begin{pmatrix} \gamma_n^\downarrow & \gamma_n^\uparrow \\ \gamma_n^\uparrow & \gamma_n^\downarrow \end{pmatrix}, \quad (54)$$

obey the useful identities

$$\underline{\Gamma}_{\parallel} \underline{K} = 0, \quad (55)$$

$$\left(\underline{\Gamma}_{\parallel} - \frac{1}{\lambda^2} D_{\parallel} \right) \underline{H} = 0, \quad (56)$$

$$\frac{2}{\lambda} D_{\parallel} (\underline{H} - \underline{K}) = \underline{1}, \quad (57)$$

which can be invoked to easily verify that the longitudinal Green's function $\underline{G}_{\parallel}(x)$ in fact fulfills Eq. (49). The real part of the complex wave vector k describes the precession of transverse nonequilibrium spin excitations while its imaginary part is the damping due to dephasing mechanisms. Therefore, the root of k has to be chosen such that it has a positive imaginary part.

In the following, we restrict ourselves to the regime $\chi \ll 1$, viz. a momentum relaxation rate much smaller than the exchange splitting. Again, since leading order correction term turns out to be of order q^2 , we drop any terms of higher order than that. Later, we will see that this restricts the validity of the result for the DWR to domain-wall lengths larger than the spin-precession length l_{prec} .

In this regime, the transverse oscillations are very rapid on the scale of the magnetization gradient. Therefore, it is suitable to eliminate the transverse degrees of freedom by first splitting the equation of motion (47) for \vec{n} into transverse and longitudinal parts,

$$(\underline{\Gamma} - D_{\parallel} \partial^2)_{\text{tt}} \vec{n}_{\text{t}} = \underline{W}_{\parallel} \vec{n}_{\parallel} + \underline{W}_{\text{tt}} \vec{n}_{\text{t}}, \quad (58)$$

$$(\underline{\Gamma} - D_{\parallel} \partial^2)_{\parallel} \vec{n}_{\parallel} = \underline{W}_{\parallel} \vec{n}_{\parallel} + \underline{W}_{\text{tt}} \vec{n}_{\text{t}}, \quad (59)$$

and writing down the formal solution for the transverse component

$$\vec{n}_{\text{t}}(x) = \int_{-\infty}^{\infty} dx' \underline{G}_{\text{tt}}(x-x') [\underline{W}_{\parallel} \vec{n}_{\parallel}(x') + \underline{W}_{\text{tt}} \vec{n}_{\text{t}}(x')]. \quad (60)$$

In the limit $\chi \ll 1$, $\underline{G}_{\text{tt}}(x')$ varies on a scale determined by $\frac{2\pi}{k} = l_{\text{prec}}$ which is much smaller than other length scales of interest, which are variation of \mathbf{m} and the external electric field. In particular, for the length of the domain wall, $w \gg l_{\text{prec}}$. Thus, to leading order in χ , we can consider $\underline{G}_{\text{tt}}$ as a representation of the Dirac δ -function and perform the integration. We obtain

$$\vec{n}_{\text{t}}(x) = \underline{\mathcal{F}} [\underline{W}_{\parallel} \vec{n}_{\parallel}(x) + \underline{W}_{\text{tt}} \vec{n}_{\text{t}}(x)]. \quad (61)$$

Here we introduced the spatially integrated transverse Green's function,

$$\underline{\mathcal{F}} = \int_{-\infty}^{\infty} dx \underline{G}_{\text{tt}}(x) = \begin{pmatrix} F & 0 \\ 0 & F^* \end{pmatrix}, \quad (62)$$

where

$$F = \frac{1}{\gamma_n^\perp} \frac{\chi \xi}{\chi \xi + i} \approx -i \frac{1}{\gamma_n^\perp} \chi \xi = -i \frac{1}{\Delta}. \quad (63)$$

Note that $\underline{\mathcal{F}}$ simply corresponds to the inverse of the transverse part of $\underline{\Gamma}$, a fact which becomes clear by noting that our approximation corresponds to the neglect of the transverse diffusion term.

Additionally, we only need to keep the first term of (61) since the backaction on the transverse dynamics, represented by the second term, appears only in higher orders in q and χ . Explicitly, this is expressed by the fact that to leading order in q , $\underline{W}_{\text{tt}}$ vanishes, so that

$$\vec{n}_{\text{t}}(x) = \underline{\mathcal{F}} \underline{W}_{\parallel} \vec{n}_{\parallel}(x) + \underline{\mathcal{F}} \underline{W}_{\text{tt}} \underline{\mathcal{F}} \underline{W}_{\parallel} \vec{n}_{\parallel}(x) + \dots = \underline{\mathcal{F}} \underline{W}_{\parallel} \vec{n}_{\parallel}(x) + O(\chi^3). \quad (64)$$

Putting this result back into the equation for the longitudinal dynamics yields the formal solution,

$$\vec{n}_{\parallel}(x) = \int_{-\infty}^{\infty} dx' \underline{G}_{\parallel}(x'-x) \times [\underline{W}_{\parallel} \vec{n}_{\parallel}(x') + \underline{W}_{\text{tt}} \underline{\mathcal{F}} \underline{W}_{\parallel} \vec{n}_{\parallel}(x')]. \quad (65)$$

The boundary condition (34) for the left and right side of the contact reads

$$\vec{n} \left(\pm \frac{L}{2} \right) = e^2 \varphi \left(\pm \frac{L}{2} \right) (0, 0, \nu_{\uparrow}, \nu_{\downarrow}). \quad (66)$$

The zeroth order solution with externally applied bias voltage V , that satisfies this boundary condition, is simply found to be

$$\vec{n}^{(0)}(x) = -e^2 E x (0, 0, \nu_{\uparrow}, \nu_{\downarrow}), \quad (67)$$

where $E = \frac{V}{L}$ is the constant external electric field in absence of the wall.

Substituting $\vec{n}^{(0)}(x)$ into the right-hand side of the solution (65) yields the second order correction in q , $\vec{n}^{(2)}(x)$. Due to charge-current conservation, the current flowing through the contact is still determined by $\vec{n}^{(0)}$ because $\partial \vec{n}^{(2)}(x)$ is taken to vanish at the boundaries by assuming that the contact is long enough for any finite size effects to become negligible, i.e., $L \gg \lambda$. In this regime, the exponential term in the longitudinal Green's function (50) can be neglected which means that spin accumulation has faded near the reservoirs. Then, the current can be deduced directly from Eqs. (67), unaffected by the correction $\vec{n}^{(2)}$. Explicitly, this current reads

$$\vec{j}^{(0)} = -\underline{D} \partial \vec{n}^{(0)} = (0, 0, \sigma_{\uparrow}, \sigma_{\downarrow})E, \quad (68)$$

where the spin-resolved Drude conductivity of majority and minority spin channels is given as usually by $\sigma_{\uparrow, \downarrow} = e^2 \nu_{\uparrow, \downarrow} D_{\uparrow, \downarrow}$.

However, $\vec{n}^{(2)}(x)$ has an additional potential drop that is extracted from the asymptotic behavior and that stems from the second term in $G_{\parallel}(x)$,

$$v_0 e^2 \delta V = n_c^{(2)}(x \rightarrow +\infty) - n_c^{(2)}(x \rightarrow -\infty) = 2n_c^{(2)}(x \rightarrow \infty). \quad (69)$$

Here, we used that the charge component $n_c = n_{\uparrow} + n_{\downarrow}$ and the second equality is due to symmetry of the contact. Keeping the current constant, the presence of the wall implies a change in the externally applied potential, which directly translates into a relative change in resistance. Hence, we define the DWR

$$\delta\rho_{\text{DW}} \equiv \frac{\rho_{\text{DW}} - \rho_0}{\rho_0} = \frac{\delta V}{V} = \frac{2n_c^{(2)}(x \rightarrow \infty)}{n_c^{(0)}(+L/2) - n_c^{(0)}(-L/2)}. \quad (70)$$

Of course, due to the perturbative nature of our treatment, the correction $n^{(2)}$ has to be always smaller than $n^{(0)}$. Since it turns out that

$$\vec{n}^{(2)}(x \rightarrow \pm\infty) = \delta\rho_{\text{DW}} \vec{n}^{(0)}\left(\pm \frac{L}{2}\right) \quad (71)$$

this condition is equivalent to $\delta\rho_{\text{DW}} \ll 1$ which is always the case.

D. Results

The correction to domain-wall resistance up to order $q^2 = [\partial \mathbf{m}(x)]^2$ takes the form

$$\delta\rho_{\text{DW}} \equiv \frac{\rho_{\text{DW}} - \rho_0}{\rho_0} = \frac{1}{3} \frac{E_{\text{DW}}}{E_{\text{F}}} f(\tilde{\Delta}, \gamma, \xi, \kappa) \quad (72)$$

where analogously to Ref. 19, we use the domain-wall energy defined by

$$f_0(\tilde{\Delta}, \gamma)$$

$$= \frac{5[2(\tilde{\Delta} - 2P) + \tilde{\Delta}^2(\tilde{\Delta} - P)] + \tilde{\Delta}\gamma(13\tilde{\Delta}(\gamma\tilde{\Delta} - \gamma^2 - 1) - 10\tilde{\Delta} + 8\gamma) - P\tilde{\Delta}\gamma[19(\gamma\tilde{\Delta} - 1 - \gamma^2\tilde{\Delta}^2) + 9(\tilde{\Delta}^2 - 3) + 4\gamma^2(1 + 2\tilde{\Delta}^2)]}{10\tilde{\Delta}}$$

$$f_{\xi}(\tilde{\Delta}, \gamma) = \frac{-(1 - \gamma^2)\tilde{\Delta}(1 - \tilde{\Delta}^2) + 2P(1 - \gamma\tilde{\Delta})(1 - 2\gamma\tilde{\Delta} + \tilde{\Delta}^2)}{\tilde{\Delta}},$$

$$f_{\kappa}(\tilde{\Delta}, \gamma) = \frac{(\tilde{\Delta} - \gamma(2 - \gamma\tilde{\Delta}))(8P - 4\tilde{\Delta} + P(-7 + P^2)\tilde{\Delta}^2 + 2\tilde{\Delta}^3)}{4(1 - \tilde{\Delta}^2)}. \quad (75)$$

$$E_{\text{DW}} = \frac{\hbar^2}{2ML} \int_{-L/2}^{+L/2} q^2(x) dx = \frac{\hbar^2}{2MwL} C. \quad (73)$$

The constant C of order unity depends on the specific form of the wall and we find the geometric scaling to be $1/wL$. For the domain-wall profile of Eq. (39) we obtain $C=2$. The scaling with $1/wL$ can be easily understood by realizing that corrections to the resistance arising from a gradient q yield the behavior $\delta\rho_{\text{DW}} \propto q^2 \propto \frac{1}{w}$. Due to physical reasons there cannot be a corrections linear in q since the result should not depend on the sign of q , i.e., the sense of rotation of \mathbf{m} . Since the total length of the contact is L and the domain wall constitutes only a fraction w/L , the correction for the whole contact should indeed be $\delta g \propto 1/wL$.

A thorough investigation of the whole hierarchy of equations reveals that the result obtained for $\delta\rho_{\text{DW}}$ is valid for wall lengths much larger than the spin-precession length, $w \gg l_{\text{prec}}$. The mathematical reason for this condition is, that, even though in the vicinity of the domain wall, the detailed profile of the quasiparticle excitations $n^{(2)}(x)$ depends on the whole hierarchy of equations (unless $w \gg l_{\text{sd}}$, with the transverse spin-diffusion length $l_{\text{sd}}^2 \equiv D_{\perp} / \gamma_{\text{n}}^{\perp}$), the asymptotic behavior of the correction $n^{(2)}(x)$, is not affected by higher order contributions of the multipole expansion. And in situations, in which finite size effects from the contact geometry are negligible, $\delta\rho_{\text{DW}}$ is solely determined by these asymptotics. In short, to obtain the asymptotics, and thus the desired result valid up to order q^2 , we need to take into account contributions up to $\vec{g}^{(4)}$.

Up to order χ^2 , the final expression for f can be split into three parts,

$$f(\tilde{\Delta}, \gamma, \xi, \kappa) = \frac{f_0(\tilde{\Delta}, \gamma) + \xi f_{\xi}(\tilde{\Delta}, \gamma) + \kappa \xi f_{\kappa}(\tilde{\Delta}, \gamma)}{\tilde{\Delta}^2(1 - \gamma^2)(1 + P(\tilde{\Delta} - \gamma) - \gamma\tilde{\Delta})}, \quad (74)$$

that take the explicit form

Let us reconsider the assumptions made during the derivation of result (72), $\chi \ll 1$ and $w \gg l_{\text{prec}}$ along with Eq. (2). The former can be rewritten as $l_s \gg l_{\text{prec}}$ where l_s denotes the scattering mean free path. This in fact implies that l_{prec} is, besides the Fermi length, the smallest length scale in the system. Note that no assumption was made on the relation between l_s and the length of the domain wall. However, we made the assumption of a diffusive contact which implies that $L \gg l_s$.

For completeness, let us also specify the leading order correction to the longitudinal current component,

$$\begin{aligned} \tilde{j}_1^{(2)} &= -e(j_s^{(2)}, -j_s^{(2)}), \\ j_s^{(2)}(x) &= eED_{\perp} v_0 \frac{1}{3} \frac{E_q(x)}{E_F} g(\tilde{\Delta}, \gamma, \xi, \kappa), \end{aligned} \quad (76)$$

which, in contrast to $\delta\rho_{\text{DW}}$, is only valid for $w \gg l_{\text{sd}}$. Obviously, due to conservation of charge current, this correction constitutes a pure spin current, $j_s^{(2)}(x)$. The x dependence is inherited trivially from $E_q(x) = \frac{q^2(x)}{2M}$.

1. Quasiclassical Regime $\tilde{\Delta} \ll 1$

Let us first investigate the limit $\Delta \ll E_F$, commonly referred to as the quasiclassical regime. However, due to the restrictions imposed upon l_{prec} and discussed above, $\tilde{\Delta}$ cannot become arbitrarily small. In this regime, corrections to the electron density of states play no role and the density of states can be considered constant. Also, it turns out that gradient corrections to the self-energies play no role so that only the contribution from impurity scattering remains. We find in this limiting case

$$f(\tilde{\Delta}, \gamma, \xi, \kappa) = \frac{\gamma^2}{1 - \gamma^2} \left(\frac{4}{5} + \xi \right) \frac{1}{\tilde{\Delta}^2}, \quad (77)$$

with the scattering asymmetry parameter γ as well as spin-dephasing strength ξ . This result shows the strong enhancement by scattering asymmetry as already noted in previous works.¹⁹ It also displays the same $\tilde{\Delta}$ dependency already found in various other works. Clearly, this result is due to different conductivities in the two spin channels which are mixed in regions of nonvanishing magnetization gradient q . A spin-up electron incident on a domain wall attains a spin-down component since electron spin direction does not instantaneously follow the local magnetization direction \mathbf{m} (known as spin mistracking). Since the electronic spectrum plays no role in this limit, any asymmetry in the conductivity of the two channels is due to the scattering asymmetry γ , thus, there is no DWR as $\gamma \rightarrow 0$.

Comparing this result with the works,^{18–20} we find in this limit, besides ξ , a different numerical prefactor. Adopting to our notation, they have $f = \frac{2}{5} \frac{\gamma^2}{1 - \gamma^2} \frac{1}{\tilde{\Delta}^2}$. To fathom this discrepancy, we stress that the specific form of the longitudinal Green's function $G_{\parallel}(x)$ is a result of the presence of spin-flip processes. In absence on these processes (which is the case in aforementioned works), the spin-diffusion length diverges, so that properly performing this limit yields the longitudinal

Green's function $G_{\parallel}(x) = H e^{-|x|/\lambda} + K \frac{|x|}{\lambda} \rightarrow (K - H) \frac{|x|}{\lambda}$, which produces qualitatively different results. This additional contribution persists arbitrarily far away from the domain wall and leads to a different result for $\delta\rho_{\text{DW}}$. Calculating then the DWR in the quasiclassical limit, we find that it is still smaller by a factor of 2 as compared to the result in.¹⁹ Nevertheless, this shows that spin-flip processes are crucial and cannot be ignored, since the absence of the latter leads to spin accumulation that does not decay even infinitely far away from the domain wall and thus yields an additional contribution to the DWR.

Concerning the work of Levy and Zhang,¹⁷ the main critic has been mentioned in the introduction. Even restricting ourselves to the quasiclassical regime, the use of a system consisting of an infinite spin-spiral the inclusion of only up to p -wave component and the lack of terms due to gauge transformation can be invoked to explain the discrepancy to our fully microscopic results.

There is still the question about how small $\tilde{\Delta}$ can become, since, clearly, the limiting factor appearing in the expressions is $1/\tilde{\Delta}^2$. However since, within our specific model,

$$\gamma \propto P(\tilde{\Delta}) = \frac{\tilde{\Delta}}{2} + O[\tilde{\Delta}]^3,$$

there is no problem concerning $\delta\rho_{\text{DW}}$, because then $\tilde{\Delta}^2$ cancels out. For the current, only $\tilde{\Delta}$ cancels, but there is another constriction leading to $E_q \ll \Delta$ which again stems from the requirement that l_{prec} is smaller than the scale of the magnetization gradient. Hence again, we will not get into trouble. This remains a problem in the work of Brataas and co-workers,¹⁹ since there, the impurity scattering times are introduced phenomenologically and thus, in their scenario no restriction is placed upon γ .

2. Arbitrary $\tilde{\Delta} < 1$

Let us now have a closer look at the behavior in the whole range of valid values for $\tilde{\Delta}$. In this regime, the gradient corrections in the collision integral become important and so is the influence of the magnetization gradient on the electronic structure, viz., the density of states.

In the half-metallic limit the spin-flip length becomes arbitrarily small, since $\lambda \rightarrow 0$ as $\tilde{\Delta} \rightarrow 1$. Writing $\Delta = E_F - \epsilon^2$ and owing to the condition that no length should exceed the precession length $l_{\text{prec}} \ll \lambda$, we obtain the requirement that $\frac{\epsilon^2}{\kappa(1-\gamma)} \gg \gamma_n^{\perp} \gamma_j^{\perp}$. However, this is not a big restriction considering our assumption that scattering is weak, so that $\gamma_j^{\perp}, \gamma_n^{\perp} \ll E_F$.

Note that within our model of impurity scattering we have only two independent parameters. A convenient choice is to vary ξ and κ independently, so that $\gamma = P(1 - \xi\kappa)$ is fixed. κ can be also regarded as magnetic scattering asymmetry parameter since we have $\kappa=0$ for $\chi_{\perp}^{(m)}=0$, $\kappa=1$ for $\chi_{\parallel}^{(m)}=\chi_{\perp}^{(m)}$ and $\kappa=2$ for $\chi_{\parallel}^{(m)}=0$. For various values of ξ and κ , the DWR is shown in Fig. 2 and the corresponding change of the spin current in the contact is depicted in Fig. 3.

We can see that the DWR does not vary strongly with Δ when this parameter is small and decreases monotonously as

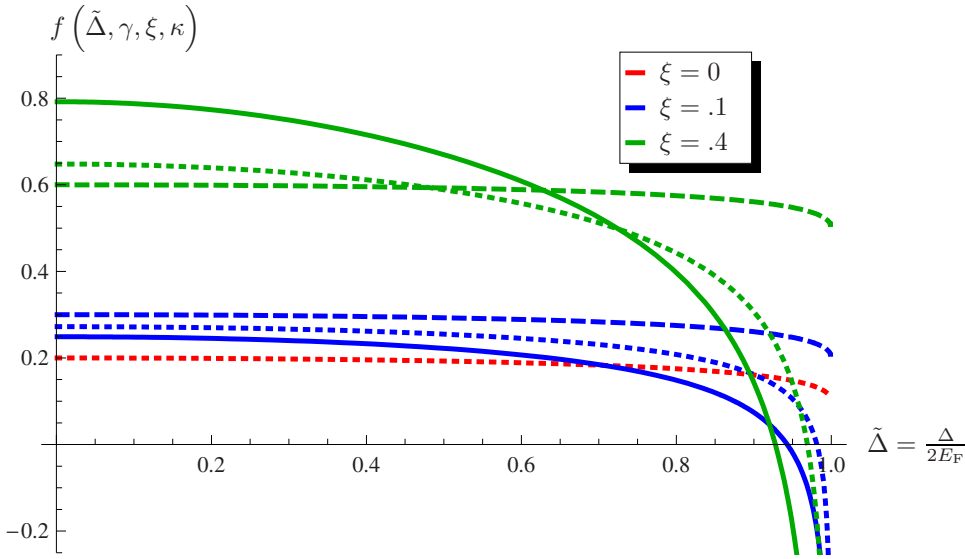


FIG. 2. (Color online) Domain-wall resistance $\delta\rho_{\text{DW}} = \frac{1}{3} \frac{E_{\text{DW}}}{E_F} f(\tilde{\Delta}, \gamma, \xi, \kappa)$ as a function of exchange splitting Δ for various values of $\xi \equiv \frac{\gamma_r}{\gamma_l}$, the ratio of spin-dephasing rate and momentum relaxation rate and denotes the transverse spin-dephasing strength. $E_{\text{DW}} = \frac{\hbar^2}{2M} \frac{2}{wL}$ is the domain-wall energy. $\kappa=1$ implies spin-isotropic magnetic scattering of strength $\chi_{\parallel}^{(m)} = \chi_{\perp}^{(m)}$ (dotted lines), while $\kappa=2$ means $\chi_{\parallel}^{(m)} = 0$ (normal lines) and $\kappa=0$ is the case of $\chi_{\perp}^{(m)} = 0$ (dashed lines). $\gamma = P(1 - \xi\kappa)$ is the scattering asymmetry parameter and is not an independent parameter in our model.

Δ increases even to the point where the DWR can become negative as one approaches the half-metallic regime. The latter happens only if $\chi_{\perp}^{(m)} \neq 0$, i.e., in the case of nonvanishing spin-flip scattering between the two bands. Also, the magnetization-gradient-induced longitudinal spin-current (c.f. Fig. 3) displays a qualitative difference between the cases where $\chi_{\perp}^{(m)}$ vanishes and where it does not. This clearly is a band structure effect and we find that this also requires to include corrections to the collision integral. It turns out that the presence of a magnetization gradient modifies the density of states so that there are corrections of order q^2 to the scattering rates that decrease the momentum relaxation rate and thus, a reduction in resistivity. In fact, the density of states for the minority spin channel strongly decreases as one approaches $\tilde{\Delta} \rightarrow 1$. Furthermore, we find that the neglect of said corrections leads to a monotonic increase of the DWR with increasing Δ as opposed to the present result. As a side note, we remark that it is important to include corrections to the collision integral up to the same order as in the approximation of the transport part of the kinetic equation for the cal-

culaton to be consistent. Finally, predictions for a given material would require realistic band structure calculations, but the present calculations demonstrate the possibility to have a negative DWR.

Let us finally compare our approach to the one of Brataas *et al.*¹⁹ by temporarily excluding spin-flip processes. In that case, we obtain for the domain-wall resistance a strictly monotonic increase with $\tilde{\Delta}$ compatible to the findings of Brataas *et al.*, in particular, there is no negative DWR. Another puzzling fact appears, when we let the difference in momentum scattering rates vanish, by setting $\gamma=0$ in our result (let us assume for a moment that γ is an independent phenomenological parameter), corresponding to $\tau_{\uparrow} = \tau_{\downarrow}$ in the result of Brataas *et al.*¹⁹ We find

$$f(\tilde{\Delta}, \gamma=0, \xi=0, \kappa=0) = \frac{\tilde{\Delta}^3(2 + \tilde{\Delta}^2) - P(4 + \tilde{\Delta}^2)}{2\tilde{\Delta}(1 + P\tilde{\Delta})} = -\frac{3}{16}\tilde{\Delta}^2 + O[\tilde{\Delta}]^4, \quad (78)$$

which yields an overall leading order term proportional to

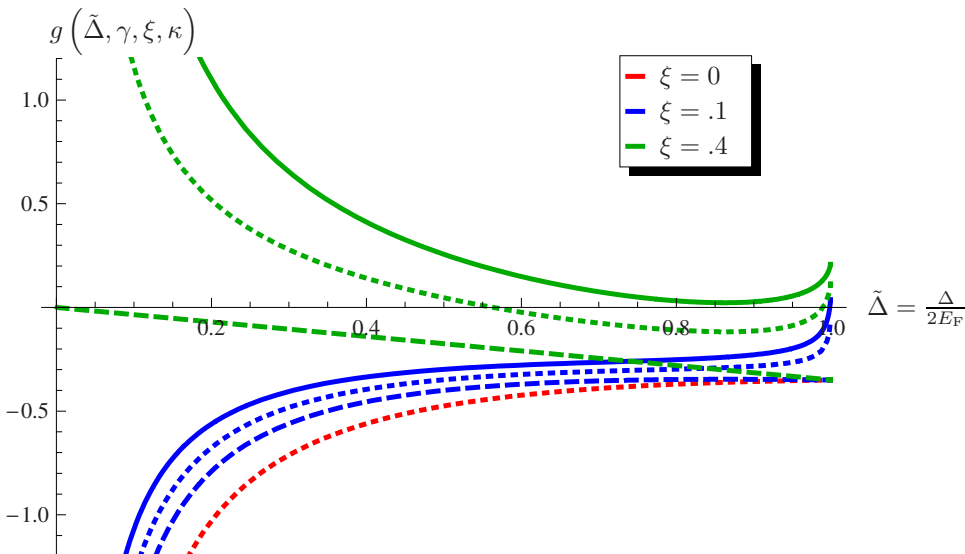


FIG. 3. (Color online) Additional magnetization-gradient-induced flow of longitudinal spin-current inside the domain wall where $E_q(x) = \frac{q^2(x)}{2M}$ does not vanish, so that $j_s^{(2)}(x) = eED_{\perp} \nu_0 \frac{1}{3} \frac{E_q(x)}{E_F} g(\tilde{\Delta}, \gamma, \xi, \kappa)$. Note that the divergence for $\tilde{\Delta} \rightarrow 0$ poses no problem, as discussed above. The magnetic scattering parameters are identical to Fig. 2.

$\tilde{\Delta}^2$. In contrast the result of Brataas *et al.* yields a constant contribution in that limit: $\delta\rho_W = \frac{3}{4} \frac{E_{DW}}{E_F}$. Interestingly, exclusion of spin-flip processes in our approach yields a result with the same asymptotics, i.e., $\tilde{f}(\tilde{\Delta}, \gamma=0, \xi=0, \kappa=0) = \frac{3}{4} + O[\tilde{\Delta}]^2$. Unsurprisingly, the results still do not coincide exactly, since we used the fully microscopic collision integral in our approach and corrections to the electronic structure are already present in this order.

IV. CONCLUSION AND OUTLOOK

We have calculated the domain-wall resistance of a Bloch wall situated in a wire of quasi-one-dimensional geometry. The current flow is perpendicular to the wall. We assumed that any spin accumulation has decayed at the contact ends, thus neglecting any finite size effects. Going toward the half-metallic regime, our calculations show the existence of a negative DWR in the presence of nonmagnetic scattering giving rise to spin-flip scattering. This possibility to obtain a negative DWR is a band structure effect. Disagreement is found when comparing our results to various previous works on DWR. We believe these discrepancies seemingly arise, on the one hand, from the neglect of spin-flip processes and, on the other hand, from the different approaches to include impurity scattering. While we use a fully microscopic approach for scattering, reflected in the full form of the collision integral (30) with gradient corrections and modification of the electronic structure properly taken into account, other works introduced momentum scattering rates phenomenologically.

To summarize, we have derived fully microscopic equation for the spin transport in non collinear magnetization textures. Our approach takes impurity scattering and spin-flip scattering into account on the Hamiltonian level. This paves the way to treat more complex magnetic textures and derive microscopic expression for the domain-wall-induced resistance.

All previous works dealing with DWR in the limit of wide walls obtain results that depend in a similar way on the microscopic parameters, that is, the DWR is $C\alpha^2$ where C is a dimensionless prefactor and $\alpha = \frac{\hbar v_F}{\Delta_w}$ is the spin mistracking angle. Hence, every theory predicts this sort of dependency, but with different proportionality factors, both in value and sign. This factor C however contains information about the scattering and DOS of the two spin channels and can depend in a complex manner on these properties. The most simple one is due to a model by Levy and Zhang¹⁷ where C simply depends on the ratio of resistivities for spin-up and spin-down channels.

We did not take into account the possibility of magnetic moment softening, i.e., the reduction of magnetic moment within the domain wall. This effect is most prominent in very sharp domain walls where canting of adjacent spin is large so that the noncollinear spin states hybridize which in turn leads to a reduction in the absolute value of the magnetic moment. As shown in Ref. 20, a reduction of the magnetic moment can lead to a negative DWR.

Finally, effects due to geometric confinement have not been considered, for example, surface scattering might be

come important. Also, the magnetization profile can be more complicated and might lead to eddy currents in the vicinity of the domain wall which might be relevant for interpretation of experimental results on the DWR in thin nanowires.

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APPENDIX: DETAILS ON DERIVING THE HIERARCHY OF EQUATIONS

We start out with the kinetic Eq. (29) and (30) by taking $(-e)\langle v_x^n(29) \rangle$ and by introducing

$$\hat{g}^{(n)} = (-e)\langle v_x^n \hat{g} \rangle = (-e) \int \frac{d^3k}{(2\pi)^3} v_x^n \hat{g},$$

$$\hat{a}^{(n)} = \langle v_x^n \hat{A} \rangle = \int \frac{d^3k}{(2\pi)^3} v_x^n \hat{A}.$$

Afterward, employing an expansion of $\circ = e^{i/2(\tilde{\partial}_x \tilde{\partial}_k - \tilde{\partial}_k \tilde{\partial}_x)} \approx 1 + \frac{i}{2}(\cdots) - \frac{1}{8}(\cdots)^2$ and making use of relation (40), we obtain

$$\begin{aligned} \left\langle v_x^n \frac{i}{2} [\hat{f}(x) \circ \hat{g}(x, k)] \right\rangle &= \frac{i}{2} [\hat{f}, \hat{g}^{(n)}] + \frac{n}{2M^2} \{ \partial \hat{f}, \hat{g}^{(n-1)} \} \\ &\quad - \frac{n(n-1)}{8M^2} \frac{i}{2} [\partial^2 \hat{f}, \hat{g}^{(n-2)}], \end{aligned} \quad (\text{A1})$$

$$\begin{aligned} \left\langle v_x^n \frac{1}{2} \{ \hat{f}(x) \circ \hat{g}(x, k) \} \right\rangle &= \frac{1}{2} \{ \hat{f}, \hat{g}^{(n)} \} - \frac{n}{2M^2} \frac{i}{2} [\partial \hat{f}, \hat{g}^{(n-1)}] \\ &\quad - \frac{n(n-1)}{8M^2} \frac{1}{2} \{ \partial^2 \hat{f}, \hat{g}^{(n-2)} \}, \end{aligned} \quad (\text{A2})$$

where we are able to truncate the series since our treatment includes only terms up to order q^2 and subsequent terms would contribute only to higher orders.

This yields the following equation:

$$\begin{aligned} \partial \hat{g}^{(n+1)} - \Delta \frac{i}{2} [\mathbf{m} \hat{\sigma}, \hat{g}^{(n)}] - \frac{\Delta n}{2M^2} \{ \partial \mathbf{m} \hat{\sigma}, \hat{g}^{(n-1)} \} \\ + \frac{\Delta n(n-1)}{8M^2} \frac{i}{2} [\partial^2 \mathbf{m} \hat{\sigma}, \hat{g}^{(n-2)}] - \frac{n}{M} (\partial \delta \mu) \hat{g}^{(n-1)} \\ = \frac{1}{2} \{ \hat{a}^{(n)}, \hat{\chi} \hat{g}^{(0)} \} - \frac{1}{2} \{ \hat{\chi} \hat{a}^{(0)}, \hat{g}^{(n)} \} + \frac{n}{2M^2} \frac{i}{2} ([\hat{a}^{(n-1)}, \partial \hat{\chi} \hat{g}^{(0)}]) \\ + [\partial \hat{\chi} \hat{a}^{(0)}, \hat{g}^{(n-1)}] - \frac{n(n-1)}{8M^2} \frac{1}{2} (\{ \hat{a}^{(n-2)}, \partial^2 \hat{\chi} \hat{g}^{(0)} \} \\ - \{ \partial^2 \hat{\chi} \hat{a}^{(0)}, \hat{g}^{(n-2)} \}), \end{aligned} \quad (\text{A3})$$

where the action of $\hat{\chi}$ is defined as

$$\hat{\chi}\hat{X} = (\chi^{(i)} + \text{Tr} \hat{\chi}^{(m)}) (\hat{X} - \xi \kappa x_{\parallel} \hat{\sigma} - \xi x_{\perp} \hat{\sigma}) = \frac{\gamma_j^{\perp}}{2\pi\nu_0} (\hat{X} - \xi \kappa x_{\parallel} \hat{\sigma} - \xi x_{\perp} \hat{\sigma})$$

and we write $x_{\parallel} = \text{Tr}\{\mathbf{m} \hat{\sigma} \hat{X}\} \mathbf{m}$ and $x_{\perp} = \text{Tr}\{\hat{\sigma} \hat{X}\} - x_{\parallel}$, so that the total impurity self-energy, consisting of spin-isotropic and magnetic parts, Eqs. (13) and (14), simply becomes

$$\check{\Sigma} = \check{\Sigma}_i + \check{\Sigma}_{\text{mag}} = \int \frac{d^3k}{(2\pi)^3} \hat{\chi} \check{G}. \quad (\text{A4})$$

We also need to know the spectral density $\hat{a}^{(n)}$ up to order q^2 . We find for even and odd moments, respectively,

$$\begin{aligned} \hat{a}^{(2n)} &= 2\pi(\nu_0^{(n)} \hat{1} + \nu_s^{(n)} \mathbf{m} \hat{\sigma}) + 2\pi(\beta_0^{(n)} \hat{1} + \beta_s^{(n)} \mathbf{m} \hat{\sigma}) (\partial \mathbf{m})^2 \\ &\quad + 2\pi\beta_{\perp}^{(n)} (\partial^2 \mathbf{m})_{\perp} \hat{\sigma}, \\ \hat{a}^{(2n+1)} &= 2\pi\alpha_{\perp}^{(n)} \mathbf{m} \times \partial \mathbf{m} \hat{\sigma}, \end{aligned} \quad (\text{A5})$$

where $(\partial^2 \mathbf{m})_{\perp}$ denotes that we only take the component perpendicular to \mathbf{m} . The coefficients for a $d=3$ dimensional electron gas are given by

$$\alpha_{\perp}^{(n)} = \frac{1}{2} \left[\frac{2}{\Delta} \nu_s^{(n+1)} - \nu_0'^{(n+1)} \right],$$

$$\beta_{\perp}^{(n)} = -\frac{1}{\Delta} \alpha_{\perp}^{(n)} - \frac{2n(2n-1)}{8M^2} \nu_s^{(n-1)},$$

$$\beta_0^{(n)} = -\frac{2n-1}{4M} \alpha_{\perp}^{(n-1)} + \frac{2n-1}{M} \nu_0^{(n-1)} \frac{\delta\mu}{q^2},$$

$$\beta_s^{(n)} = -\frac{1}{2\Delta} \alpha_{\perp}^{(n)} - \frac{2n-1}{8M} \nu_s^{(n-1)} \left[1 - \frac{8M}{q^2} \delta\mu \right],$$

using the definitions

$$\nu_0^{(n)}(\omega) = \frac{1}{2} [\nu_{\uparrow}^{(n)}(\omega) + \nu_{\downarrow}^{(n)}(\omega)],$$

$$\nu_s^{(n)}(\omega) = \frac{1}{2} [\nu_{\uparrow}^{(n)}(\omega) - \nu_{\downarrow}^{(n)}(\omega)],$$

$$\nu_{\uparrow,\downarrow}^{(n)}(\omega) = \frac{\nu_{\uparrow,\downarrow}(\omega)}{1+2n} \left[\frac{2}{M} \left(\omega \pm \frac{\Delta}{2} \right) \right]^n.$$

$\nu_{\uparrow,\downarrow}$ specified without any argument implies that we take its

value at the Fermi level, i.e., more specifically $\nu_{\uparrow,\downarrow}^{(n)} \equiv \nu_{\uparrow,\downarrow}^{(n)}(E_F)$.

The chemical potential $\delta\mu$ is obtained from the condition that

$$\begin{aligned} &\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} f_D(\omega) \text{Tr}\{\hat{a}^{(0)}(\omega) - \hat{a}^{(0)}(\omega)|_{q=0}\} \\ &= \int_{-\infty}^{+\infty} d\omega f_D(\omega) 2\beta_0^{(0)} q^2 = \int_{-\infty}^{+\infty} d\omega f_D(\omega) \frac{2}{M} \left(\frac{1}{4} \alpha_{\perp}^{(-1)} q^2 \right. \\ &\quad \left. - \nu_0^{(-1)} \delta\mu \right) = \frac{1}{2} \alpha_{\perp}^{(0)} q^2 + 2\nu_0^{(0)} \delta\mu \stackrel{!}{=} 0, \end{aligned} \quad (\text{A6})$$

since in the zero-temperature approximation

$$\int_{-\infty}^{+\infty} d\omega f_D(\omega) \nu_{\uparrow,\downarrow}^{(n)}(\omega) = \int^{E_F} d\omega \nu_{\uparrow,\downarrow}^{(n)}(\omega) = \frac{M}{2n+1} \nu_{\uparrow,\downarrow}^{(n+1)}.$$

Therefore, we immediately arrive at

$$\delta\mu = -\frac{1}{4\nu_0^{(0)}} q^2 \alpha_{\perp}^{(0)} = \frac{q^2}{2M} \frac{1}{4} \left(1 - \frac{2M\nu_s^{(1)}}{\Delta \nu_0^{(0)}} \right), \quad (\text{A7})$$

which yields Eq. (22), once we plug in all definitions and by noting that $\nu_0^{(0)} = \nu_0$, $\nu_s^{(0)} = P\nu_0$ and $\nu_s^{(1)} = \frac{\nu_0}{3M}(2PE_F + \Delta)$. Substituting $\delta\mu$ back into the expressions for β , we can now write

$$\beta_0^{(n)} = -\frac{2n-1}{4M} \alpha_{\perp}^{(n-1)} - \frac{\nu_0^{(n)}}{4\nu_0^{(0)}} \alpha_{\perp}^{(0)},$$

$$\beta_s^{(n)} = -\frac{1}{2\Delta} \alpha_{\perp}^{(n)} - \frac{1}{4\Delta} \frac{\nu_s^{(1)}}{\nu_0^{(0)}} \nu_s'^{(n)}.$$

Later, we will also need

$$\begin{aligned} \hat{\chi}\hat{a}^{(0)} &= \hat{\chi}\hat{a}_0^{(0)} + \hat{\chi}\hat{a}_q^{(0)} \rightarrow \gamma_j^{\perp} (\hat{1} + P(1 - \xi\kappa)\mathbf{m} \hat{\sigma}) + \frac{\gamma_j^{\perp}}{\nu_0} (\beta_0^{(0)} \hat{1} \\ &\quad + \beta_s^{(0)} (1 - \xi\kappa)\mathbf{m} \hat{\sigma}) q^2 + \frac{\gamma_j^{\perp}}{\nu_0} \beta_{\perp}^{(0)} (1 - \xi) (\partial^2 \mathbf{m})_{\perp} \hat{\sigma}, \end{aligned}$$

where we write $\hat{a}_0^{(0)}$ to indicate that we take the zeroth order in q only and accordingly, $\hat{a}_q^{(0)}$ contains all corrections to the density of states due to a magnetization gradient.

Next, we will change the 2×2 spin-matrix representation to the 4×4 matrix representation in the basis introduced above, $(+, -, \uparrow, \downarrow)$. This change of basis implies a local rotation of the basis in spin space to align the magnetization direction \mathbf{m} along the new z axis, which corresponds to the gauge transformation introduced above. The various vectors

we will encounter in the following derivation transform in the following manner:

$$\mathbf{m}\hat{\boldsymbol{\sigma}} \rightarrow \hat{\boldsymbol{\sigma}}_z,$$

$$\partial\mathbf{m}\hat{\boldsymbol{\sigma}} \rightarrow q\hat{\boldsymbol{\sigma}}_y,$$

$$\mathbf{m} \times \partial\mathbf{m} \rightarrow -q\boldsymbol{\sigma}_x,$$

$$\partial^2\mathbf{m}\hat{\boldsymbol{\sigma}} = (\partial^2\mathbf{m})_{\perp}\hat{\boldsymbol{\sigma}} + (\mathbf{m}\partial^2\mathbf{m})\mathbf{m}\hat{\boldsymbol{\sigma}} \rightarrow (\partial q)\hat{\boldsymbol{\sigma}}_y - q^2\hat{\boldsymbol{\sigma}}_z,$$

and we note that $-\mathbf{m}\partial^2\mathbf{m} = (\partial\mathbf{m})^2 = q^2$. In the following, we specify substitution rules that perform this transformation:

$$\frac{i}{2}[\hat{\boldsymbol{\sigma}}_i, \hat{n}] \rightarrow \underline{M}_i \vec{n},$$

$$\frac{1}{2}\{\hat{\boldsymbol{\sigma}}_i, \hat{n}\} \rightarrow \underline{K}_i \vec{n}, \quad (\text{A8})$$

where

$$\underline{M}_x = \frac{i}{2} \begin{pmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix}, \quad \underline{M}_y = \frac{1}{2} \begin{pmatrix} 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & 1 \\ 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \end{pmatrix}, \quad \underline{M}_z = \begin{pmatrix} -i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\underline{K}_x = \frac{1}{2} \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}, \quad \underline{K}_y = \frac{i}{2} \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & -1 \\ -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix}, \quad \underline{K}_z = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

constitute matrices in $(+, -, \uparrow, \downarrow)$ representation. The action of $\hat{\chi}$ turns into

$$\hat{\chi}\hat{n} \rightarrow \frac{\gamma_j^{\perp}}{2\pi\nu_0} \chi \vec{n}$$

with the matrix

$$\chi = \underline{1} + \xi \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -\frac{\kappa}{2} & \frac{\kappa}{2} \\ 0 & 0 & \frac{\kappa}{2} & -\frac{\kappa}{2} \end{pmatrix}.$$

As a consequence of the gauge transformation, the derivative transforms into

$$\partial \rightarrow \underline{\partial} \equiv \underline{1} \partial + q \underline{M}_x.$$

Now with these rules at hand, the change of representation is straightforward and we obtain

$$\underline{\partial} \vec{g}^{(n+1)} + \underline{\Pi} \vec{g}^{(n)} = \underline{\Xi}_0^{(n)} \vec{g}^{(0)} + \underline{\Xi}_q^{(n)} \vec{g}^{(0)} - \underline{\Pi}_q^{(n)} \vec{g}^{(n)} + \underline{\zeta}_1^{(n)} \vec{g}^{(n-1)} + \underline{\zeta}_2^{(n)} \vec{g}^{(n-2)}, \quad (\text{A9})$$

where zeroth order relaxation and precession terms are

$$\frac{1}{2}\{\hat{\chi}\hat{a}_0^{(0)}, \hat{g}^{(n)}\} - \Delta \frac{i}{2}[\mathbf{m}\hat{\boldsymbol{\sigma}}, \hat{g}^{(n)}] \rightarrow [\gamma_j^{\perp}(\underline{1} + P(1 - \xi\kappa)\underline{K}_z) - \Delta \underline{M}_z] \vec{g}^{(n)} \equiv \underline{\Pi} \vec{g}^{(n)} \quad (\text{A10})$$

and magnetization-gradient correction to relaxation rates yield

$$\frac{1}{2}\{\hat{\chi}\hat{a}_q^{(0)}, \hat{g}^{(n)}\} \rightarrow \frac{\gamma_j^{\perp}}{\nu_0} [q^2 \beta_0^{(0)} \underline{1} + q^2 \beta_s^{(0)} (1 - \xi\kappa) \underline{K}_z + (\partial q) \beta_{\perp}^{(0)} (1 - \xi) \underline{K}_y] \vec{g}^{(n)} \equiv \underline{\Pi}_q^{(n)} \vec{g}^{(n)}. \quad (\text{A11})$$

Corrections that depend on lower moments $\vec{g}^{(n)}$ in the hierarchy read explicitly

$$\frac{\Delta n}{2M} \frac{1}{2} \{\partial\mathbf{m}\hat{\boldsymbol{\sigma}}, \hat{g}^{(n-1)}\} + \frac{n}{M} (\partial\delta\mu) \hat{g}^{(n-1)} + \frac{n}{2M} \frac{i}{2} [\partial\hat{\chi}\hat{a}^{(0)}, \hat{g}^{(n-1)}] \rightarrow \left[q \frac{\Delta n}{2M} \underline{K}_y + \frac{n}{M} (\partial\delta\mu) \underline{1} + q \frac{n}{2M} \gamma_j^{\perp} \gamma \underline{M}_y \right] \vec{g}^{(n-1)} \equiv \underline{\zeta}_1^{(n)} \vec{g}^{(n-1)} \quad (\text{A12})$$

and

$$\begin{aligned}
 & -\frac{\Delta n(n-1)}{8M^2} \frac{i}{2} [\partial^2 \mathbf{m} \hat{\sigma}, \hat{g}^{(n-2)}] + \frac{n(n-1)}{8M^2} \frac{1}{2} \{\partial^2 \hat{\chi} \hat{a}^{(0)}, \hat{g}^{(n-2)}\} \\
 & \rightarrow \frac{n(n-1)}{8M^2} [\Delta(q^2 \underline{M}_z - (\partial q) \underline{M}_y) - \gamma_j^\perp \gamma(q^2 \underline{K}_z \\
 & - (\partial q) \underline{K}_y)] \hat{g}^{(n-2)} \equiv \zeta_2^{(n)} \hat{g}^{(n-2)}. \quad (\text{A13})
 \end{aligned}$$

Furthermore, we have source terms appearing in the equation, whereof the zeroth order term simply is

$$\frac{1}{2} \{\hat{a}_0^{(2n)}, \hat{\chi} \hat{g}^{(0)}\} \rightarrow \frac{\gamma_j^\perp}{\nu_0} [\nu_0^{(n)} \underline{1} + \nu_s^{(n)} \underline{K}_z] \chi \hat{g}^{(0)} \equiv \Xi_0^{(2n)} \hat{g}^{(0)} \quad (\text{A14})$$

for even indices and $\Xi_0^{(2n+1)}=0$ for odd indices. The corresponding gradient corrections are, for even indices

$$\begin{aligned}
 & \frac{1}{2} \{\hat{a}_q^{(2n)}, \hat{\chi} \hat{g}^{(0)}\} + \frac{2n}{2M} \frac{i}{2} [\hat{a}^{(2n-1)}, \partial \hat{\chi} \hat{g}^{(0)}] \\
 & - \frac{2n(2n-1)}{8M^2} \frac{1}{2} \{\hat{a}^{(2n-2)}, \partial^2 \hat{\chi} \hat{g}^{(0)}\} \rightarrow \frac{\gamma_j^\perp}{\nu_0} \left[q^2 \beta_0^{(n)} \underline{1} \right. \\
 & + q^2 \beta_s^{(n)} \underline{K}_z + (\partial q) \beta_\perp^{(n)} \underline{K}_y - q \frac{2n}{2M} \alpha_\perp^{(n-1)} \underline{M}_x \partial \\
 & \left. - \frac{2n(2n-1)}{8M^2} (\nu_0^{(n-1)} \underline{1} + \nu_s^{(n-1)} \underline{K}_z) \partial^2 \right] \chi \hat{g}^{(0)} \equiv \Xi_q^{(2n)} \hat{g}^{(0)}. \quad (\text{A15})
 \end{aligned}$$

and for odd indices

$$\begin{aligned}
 & \frac{1}{2} \{\hat{a}_q^{(2n+1)}, \hat{\chi} \hat{g}^{(0)}\} + \frac{2n+1}{2M} \frac{i}{2} [\hat{a}^{(2n)}, \partial \hat{\chi} \hat{g}^{(0)}] \rightarrow \frac{\gamma_j^\perp}{\nu_0} \left[-q \alpha_\perp^{(n)} \underline{K}_x \right. \\
 & \left. + \frac{2n+1}{2M} \nu_s^{(n)} \underline{M}_z \partial \right] \chi \hat{g}^{(0)} \equiv \Xi_q^{(2n+1)} \hat{g}^{(0)}. \quad (\text{A16})
 \end{aligned}$$

For our purpose, we only need the first five equations, since, as stated previously, in our regime of investigation we need to know only up to $\hat{g}^{(4)}$. Explicitly, these equations read

$$\partial \vec{j} + (\underline{\Pi} - \Xi_0^{(0)}) \vec{n} = (\Xi_q^{(0)} - \underline{\Pi}_q^{(0)}) \vec{n}, \quad (\text{A17})$$

$$\partial \vec{S} + \underline{\Pi} \vec{j} = \Xi_q^{(1)} \vec{n} - \underline{\Pi}_q^{(1)} \vec{j} + \zeta_1^{(1)} \vec{n}, \quad (\text{A18})$$

$$\partial \vec{T} + \underline{\Pi} \vec{S} - \Xi_0^{(0)} \vec{n} = \Xi_q^{(2)} \vec{n} - \underline{\Pi}_q^{(2)} \vec{S} + \zeta_1^{(2)} \vec{j} + \zeta_2^{(2)} \vec{n}, \quad (\text{A19})$$

$$\partial \vec{U} + \underline{\Pi} \vec{T} = \Xi_q^{(3)} \vec{n} + \zeta_1^{(3)} \vec{S} + O(q^3), \quad (\text{A20})$$

$$\underline{\Pi} \vec{U} - \Xi_0^{(4)} \vec{n} = O(q), \quad (\text{A21})$$

where we defined $\vec{n} = \hat{g}^{(0)}$, $\vec{j} = \hat{g}^{(1)}$, $\vec{S} = \hat{g}^{(2)}$, $\vec{T} = \hat{g}^{(3)}$, and $\vec{U} = \hat{g}^{(4)}$. Here, we already dropped terms that would only contribute to higher orders than q^2 . Note that $\Gamma = \underline{\Pi} - \Xi_0^{(0)}$.

Our aim is to obtain a differential equation of the form (47).

$$(\underline{\Gamma} - \underline{D} \partial^2) \vec{n} = \underline{W}(q) \vec{n}, \quad (\text{A22})$$

where

$$\begin{aligned}
 \underline{W}(q) = & \underline{Y}_{q\partial} q \partial + \underline{Y}_{\partial q} \partial q + \underline{Y}_{\partial q \partial} \partial q^2 \partial + \underline{Y}_{q\partial \partial} q \partial q \partial \\
 & + \underline{Y}_{\partial q \partial} \partial q \partial q. \quad (\text{A23})
 \end{aligned}$$

To achieve this, we unite the set of Eqs. (A17)–(A21) iteratively by eliminating every moment except \vec{n} . We do this order by order in q, ∂ and in the following, we give only terms that are relevant to our result. The zeroth order is simply $\vec{S}_0 = \underline{\Pi}^{-1} \Xi_0^{(2)} \vec{n}$ and $\vec{U}_0 = \underline{\Pi}^{-1} \Xi_0^{(4)} \vec{n}$ while the odd moments \vec{j}, \vec{T} vanish as $q, \partial \rightarrow 0$. \vec{S}_0 , when plugged into Eq. (A18) will yield a term that resembles a diffusion term, but will not yet be of the desired form as given by Eq. (46). In order to transform the diffusion term into the form which permits convenient solution of the differential equation, we will multiply Eq. (A22) to the left with $\underline{\Pi}_q \underline{A}$ and then subtract it from the Eq. (A19) which provides \vec{S} . We thus obtain a new $\vec{S}_0 = (\underline{\Pi}^{-1} \Xi_0^{(2)} + \underline{A} \underline{\Gamma}) \vec{n}$ with the choice of \underline{A} determined by the condition that $\vec{S}_0 = \underline{\Pi} \underline{D} \vec{n}$ which in turn translates into

$$\underline{A} = (\underline{\Pi} \underline{D} - \underline{\Pi}^{-1} \Xi_0^{(2)}) \underline{\Gamma}^{-1} \quad (\text{A24})$$

and, since $\underline{\Gamma}$ is singular in longitudinal subspace as it possesses one vanishing eigenvalue, $\underline{\Gamma}^{-1}$ has to be considered as the pseudo inverse.

In order to obtain the form reminiscent of the spin-charge diffusion equation (41), we rewrite Eq. (A18).

$$\underline{\Pi} (\vec{j} + \underline{D} \partial \vec{n}) = \underline{\Pi} \underline{D} \partial \vec{n} - \partial \vec{S} + \Xi_q^{(1)} \vec{n} - \underline{\Pi}_q^{(1)} \vec{j} + \zeta_1^{(1)} \vec{n}, \quad (\text{A25})$$

where now the right-hand side of this equation is of order q , in particular the difference $\underline{\Pi} \underline{D} \vec{n} - \vec{S}$ no longer contributes to the zeroth order solution, as opposed to \vec{S} itself.

Before proceeding with higher orders, we define for convenience.

$$\underline{\Pi}_q^{(n)} = \underline{\Pi}_{qq}^{(n)} q^2 + \underline{\Pi}_{\partial q}^{(n)} (\partial q - \partial q),$$

$$\zeta_1^{(n)} = \zeta_{1,q}^{(n)} q + \zeta_{1,\partial q}^{(n)} (\partial q^2 - q^2 \partial),$$

$$\zeta_2^{(n)} = \zeta_{2,qq}^{(n)} q^2 + \zeta_{2,\partial q}^{(n)} (\partial q - \partial q),$$

$$\Xi_q^{(2n)} = \Xi_{qq}^{(2n)} q^2 + \Xi_{\partial q}^{(2n)} \partial q + \Xi_{q\partial}^{(2n)} q \partial,$$

$$\Xi_q^{(2n+1)} = \Xi_q^{(2n+1)} q + \Xi_\partial^{(2n+1)} \partial,$$

where we remind that ∂ acts on everything to its right so that Ξ_q actually are differential operators.

The first order is given by $[\vec{j} = (\underline{\mathcal{J}}_q q + \underline{\mathcal{J}}_\partial \partial) \vec{n}$, etc.]

$$\underline{\mathcal{J}}_q = -\underline{\Pi}^{-1} (\underline{M}_x \underline{\Pi} \underline{D} - \Xi_q^{(1)} - \zeta_1^{(1)}),$$

$$\underline{\mathcal{J}}_\partial = -\underline{D} + \underline{\Pi}^{-1} \Xi_\partial^{(1)},$$

$$\underline{\mathcal{T}}_q = -\underline{\Pi}^{-1} (\underline{M}_x \underline{\Pi}^{-1} \Xi_0^{(4)} - \Xi_q^{(3)} - \zeta_1^{(3)} \underline{\Pi} \underline{D}),$$

$$\underline{\mathcal{T}}_{\partial} = -\underline{\Pi}^{-1}(\underline{\Pi}^{-1}\underline{\Xi}_{\partial}^{(4)} - \underline{\Xi}_{\partial}^{(3)}),$$

the second order terms are

$$\underline{Y}_{\partial q} = \underline{\Pi}^{-1}\underline{M}_x\underline{\Pi}D + \underline{\Xi}_{\partial q}^{(0)} - \underline{\Pi}_{\partial q}^{(0)} - \underline{\Pi}^{-1}(\underline{\Xi}_q^{(1)} + \underline{\zeta}_{1q}^{(1)}),$$

$$\underline{Y}_{q\partial} = \underline{M}_x\underline{D} + \underline{\Xi}_{q\partial}^{(0)} + \underline{\Pi}_{q\partial}^{(0)} - \underline{M}_x\underline{\Pi}^{-1}\underline{\Xi}_{\partial}^{(1)},$$

$$\underline{S}_{\partial q} = \underline{\Pi}^{-1}[-\underline{\mathcal{T}}_q - \underline{\Pi}_{\partial q}^{(2)}\underline{\Pi}D + (\underline{\Xi}_{\partial q}^{(2)} + \underline{\zeta}_{2\partial q}^{(2)})] - \underline{A}Y_{\partial q},$$

$$\underline{S}_{q\partial} = \underline{\Pi}^{-1}[-\underline{M}_x\underline{\mathcal{T}}_{\partial} + \underline{\Pi}_{\partial q}^{(2)}\underline{\Pi}D + \underline{\zeta}_{1q}^{(2)}\underline{\mathcal{T}}_{\partial} + (\underline{\Xi}_{q\partial}^{(2)} - \underline{\zeta}_{2\partial q}^{(2)})] - \underline{A}Y_{q\partial},$$

and finally, the 3 remaining coefficients in Eq. (A23),

$$\underline{Y}_{\partial q\partial} = \underline{\Pi}^{-1}(\underline{M}_x\underline{S}_{q\partial} + \underline{\Pi}_{qq}^{(1)}\underline{\mathcal{T}}_{\partial} + \underline{\zeta}_{1\partial qq}^{(1)}),$$

$$\underline{Y}_{q\partial q\partial} = \underline{M}_x\underline{\Pi}^{-1}(\underline{S}_{q\partial} + \underline{\Pi}_{\partial q}^{(1)}\underline{\mathcal{T}}_{\partial}),$$

$$\underline{Y}_{\partial q\partial q} = \underline{\Pi}^{-1}(\underline{M}_x\underline{S}_{\partial q} - \underline{\Pi}_{\partial q}^{(1)}\underline{\mathcal{T}}_q).$$

To solve Eq. (A22), we use the method elaborated upon previously in this paper, so that by use of Eqs. (65) and (70) we end up with

$$\delta\rho_{\text{DW}} = \left[\frac{K}{\lambda} [2(\underline{Y}_{\partial q})_{\text{ll}}\underline{\mathcal{F}}(\underline{Y}_{q\partial})_{\text{tl}} + (\underline{Y}_{\partial q})_{\text{ll}}\underline{\mathcal{F}}(\underline{Y}_{\partial q})_{\text{tl}} + (2\underline{Y}_{\partial q\partial} + \underline{Y}_{q\partial q\partial} + \underline{Y}_{\partial q\partial q})_{\text{ll}}]\vec{n}_0 \right]_{\text{c}}, \quad (\text{A26})$$

where the outer bracket denotes that we take the charge component.

*christian.wickles@uni-konstanz.de

†wolfgang.belzig@uni-konstanz.de

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